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A Decision Support Method for Facility Location Planning in Population Decrease

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Abstract: Population decrease requires the reconstruction of urban structure. Closure and relocation of urban facilities are indispensable for achieving efficient and sustainable cities. This paper proposes a new decision support method for facility location planning. Facility location problem is formulated as a spatial optimization problem. Multiple plans are derived by solving the problem. The result is quantitatively evaluated by numerical measures and graphically visualized by maps. Interpretation and analysis of the current situation and derived plans permits us to discuss a wide variety of facility plans such as facility relocation, facility expansion, development of complex facilities and improvement of transport accessibility. The method is applied to a location planning of elementary schools in Chiba City, Japan. It illustrates a concrete usage of the method proposed as well as provides empirical findings.

Keywords: Decision support, facility location, spatial optimization

1. Introduction

Population decrease is one of the most critical problems in developed countries. Low birth rate and the aging of society have a great impact on labor market and medical insurance system. In suburban and rural areas it is often almost impossible to keep the local community and public services.

Drastic change in population structure requires urban reconstruction (Register, 2006; Farr, 2007; Langner and Endlicher, 2007). While elementary schools serve fewer pupils than they have capacity for, welfare facilities for old people are not enough to meet their demand. To keep moderate city size and population density is essential to resolve environmental issues and improve economic efficiency.

Among those issues this paper deals with the facility location planning in population decrease. Some facilities have to be closed or converted into another use for economic efficiency. Since it greatly affects the local community, decision making requires a detailed and full discussion from both qualitative and quantitative points of view.

In facility location planning, spatial optimization is a useful tool that provides a plan of facility location that is optimal in a certain aspect (Mirchandani and Francis, 1990; Daskin, 1995; Drezner, 1995; Drezner and Hamacher, 2004). Though the optimization of facility location involves many hypotheses on users' behavior and transport environment, it is still useful as a draft proposal that serves as material for discussion. Nevertheless, spatial optimization is not so often used in urban planning. One reason is that urban planning is substantially an exploratory and collaborative process toward a decision making (Healey, 1997; Saaty and Peniwati, 2007). A wide variety of plans are considered and evaluated simultaneously from various aspects. New plans and policies emerge during planning process which are then compared with earlier ones. In location planning, for instance, policies of cost reduction other than facility closure is discussed such as the reduction of opening hours and the introduction of automated systems. Different policies may be adopted among different regions. Since spatial optimization assumes a highly abstract model of the real world, it lacks of flexibility that is indispensable to treat a wide variety of values.

One method to resolve this problem is to use multiobjective optimization technique (Steuer, 1986; Ehrgott, 2000; Deb, 2001; Belton and Stewart, 2002; Figueira *et al*., 2004; Branke *et al*., 2008). Once a mathematical optimization problem is formulated, powerful techniques such as evolutionary algorithms can solve it in a practical time. This approach, however, is not enough to treat the flexibility of urban planning. Since the optimization problem is an abstract model of the real world, interpretation and analysis of the result are indispensable in its practical use to fill the gap between the model and the real world.

To this end, this paper proposes a new decision support method for facility location planning. Though it is based on spatial optimization technique, it permits more flexible discussion during planning process. Location planning focuses on a typical type of urban facilities whose maximum capacity and service distance are given as constraints. The method permits us to consider not only the closure of existing facilities but also other options such as the conversion into different uses and the development of multiuse facilities.

Section 2 proposes a decision support method for facility location planning. Location planning is formulated as a mathematical optimization problem. Multiple plans are derived and evaluated by numerical measures. To discuss the benefits and limitations of the method, Section 3 applies it to school location planning in Japan. Section 4 summarizes the conclusions with a discussion.

2. Decision support method

This section proposes a decision support method for facility location planning in population decrease. It consists of four steps: 1) Formulation and an initial solution of facility location problem, 2) generation of alternative solutions, 3) analysis of current situation, and 4) analysis of alternative set. They are described successively in the following.

2.1 Formulation and an initial solution of facility location problem

Consider a set of existing facilities $\Lambda=\{P_i,\ i\in[\mathcal{M}]\}$ ($[\mathcal{M}]=\{1, 2, ..., M\}$) and their users $\Omega=\{Q_j,\ j\in[\mathcal{N}]\}$ ($[\mathcal{N}]=\{1, 2, ..., N\}$) in a two-dimensional region S. Distance between facility P_i and user Q_j is given by d_{ij}. Maximum capacity of P_i and maximum service distance are denoted by C_i and d_{max}, respectively. Users living outside the service area of existing facilities are out of scope of this paper.

Every user in the service area of facilities is assumed to be allocated to one of the facilities. Facility location problem is then formulated as a set covering problem where the number of facilities should be minimized. Let x_i and y_{ij} be binary functions that indicate whether P_i remains or closes and whether Q_i if allocated to P_i, respectively. The problem to be solved is then defined as:

Problem M$_1$:

$$\min_{x_i, y_{ij}, i\in\mathfrak{m}, j\in\mathfrak{n}} \sum_{i\in\mathfrak{m}} x_i \ , \tag{1}$$

subject to

$$\begin{aligned} x_i \sum_{j\in\mathfrak{n}} y_{ij} &\le C_i \quad (\forall i \in \mathfrak{m}) \\ y_{ij} d_{ij} &\le d_{\max} \quad (\forall i \in \mathfrak{m}\ \forall j \in \mathfrak{n}) . \\ \forall j, \sum_{i\in\mathfrak{m}} y_{ij} &= 1 \quad (\forall j \in \mathfrak{n}) \end{aligned} \tag{2}$$

The first and second constraints are capacity and distance constraints, respectively.

Since Problem M$_1$ is an integer problem, it should be solved by using a heuristic approach. A solution is denoted by $S_1=\{s_{11}, s_{12}, ..., s_{1m}\}$, where s_{1i} is a binary function given by $s_{1i}=x_i$. The minimum number of facilities is denoted by $M_{\min}$ (= $\sum_{i\in\mathfrak{m}} s_{1i}$).

2.2 Generation of alternative solutions

Initial solution S_1 gives both the minimum number of facilities and the set of facilities to be retained. The former, however, can be usually realized by different sets of facilities if the constraints are not too restrictive. To extend the variety of alternatives, this section derives another solutions that also minimizes the number of facilities.

To obtain an alternative as much different as possible from S_1, we add S_1 as a penalty function in the objective function:

Problem M$_2$:

$$\min_{x_i, y_{ij}, i\in\mathfrak{m}, j\in\mathfrak{n}} \sum_{i\in\mathfrak{m}} x_i + \lambda \sum_{i\in\mathfrak{m}} s_{1i} \,, \tag{3}$$

subject to

$$\begin{aligned} x_i \sum_{j\in\mathfrak{n}} y_{ij} &\le C_i \quad (\forall i \in \mathfrak{m}) \\ y_{ij} d_{ij} &\le d_{\max} \quad (\forall i \in \mathfrak{m}\ \forall j \in \mathfrak{n}), \\ \forall j, \sum_{i\in\mathfrak{m}} y_{ij} &= 1 \quad (\forall j \in \mathfrak{n}) \end{aligned} \tag{4}$$

where λ is a constant parameter. Since Problem M_2 contains S_1 as a solution that should be avoided, it is expected to give a solution different from S_1.

If the solution of Problem M_2 also gives the same number of facilities as $M_{\min}$, the solution is denoted by $S_2=\{s_{21}, s_{22}, ..., s_{2m}\}$. The same process is repeated until L_1 solutions are obtained. Problem M_k is represented as

Problem M_k:

$$\min_{x_i, y_{ij}, i\in\mathfrak{m}, j\in\mathfrak{n}} \sum_{i\in\mathfrak{m}} x_i + \lambda \sum_{i\in\mathfrak{m}} s_{k-1,i} \,, \tag{5}$$

subject to

$$\begin{aligned} x_i \sum_{j\in\mathfrak{n}} y_{ij} &\le C_i \quad (\forall i \in \mathfrak{m}) \\ y_{ij} d_{ij} &\le d_{\max} \quad (\forall i \in \mathfrak{m}\ \forall j \in \mathfrak{n}). \\ \forall j, \sum_{i\in\mathfrak{m}} y_{ij} &= 1 \quad (\forall j \in \mathfrak{n}) \end{aligned} \tag{6}$$

From L solutions $\{S_1, S_2, ..., S_{L1}\}$ we remove duplicate ones and renumber the remaining sets to fill the missing ones. As a result we obtain the first set of solutions $\Psi=\{S_1, ..., S_{L2}\}$, where $S_k=\{s_{k1}, s_{k2}, ..., s_{km}\}$.

We derive further alternatives from Ψ by exchanging two elements in each solution in Ψ. In S_k, for instance, we choose a pair of elements whose values are zero and one, respectively. We switch their values and add the new set to Ψ if it satisfies the given constraints and different from those in Ψ. We repeat this process until enough number of different alternatives are obtained. The final set of solutions is denoted by $\Psi_F=\{S_1, ..., S_T\}$, where T is the number of necessary alternatives that is given a priori.

2.3 Analysis of the current situation

Having obtained the final set of solutions Ψ_F, we analyze and evaluate them from various points of view. However, it clearly requires us to understand the current situation of the study area. This section propose several measures to analyze the strength of constraints at both local and global scales.

Whether a facility is necessary or not depends on distance and capacity constraints. The former requires facilities to remain in rural areas even if they have only a few users. In urban areas, on the other hand, facilities have to be densely located to serve users at their maximum capacity.

Let us consider the situation where the two constraints are in equilibrium. If facility users are uniformly distributed, it emerges when users of the maximum capacity exist in the service area of a facility. Mathematically it is represented as

$$C_i = \pi d_{\max}^2 \gamma \,, \tag{7}$$

where γ is the average density of facility users. Assuming all the facilities have the same capacity C, we have

$$\gamma = \frac{C}{\pi d_{\max}^2}. \tag{8}$$

The density γ is called *equilibrium density*. It is a useful global measure to discuss roughly which constraint is more restrictive by using this measure. The measure is effective especially when comparing the facility location across different regions.

For more detailed discussion we propose four measures in the following. Let σ_{ij} be a binary function indicating whether facility P_i is accessible from user Q_i:

$$\sigma_{ij} = \begin{cases} 1 & if\ d_{ij} \le d_{\max} \\ 0 & \text{otherwise} \end{cases}. \tag{9}$$

Distance constraint is restrictive for users if they have only few facilities within the maximum service distance. We thus define *distance constraint measure of user* Q_i as the inverse of the number of accessible facilities:

$$\lambda_D\left(Q_j\right) = \frac{1}{\sum_i \sigma_{ij}}. \tag{10}$$

Let us assume that every facility user is randomly allocated to accessible facilities. The probability is given by $\lambda_D(Q_i)$ defined in Equation (10). Expected number of users of facility P_i is then given by

$$C'\left(P_i\right) = \sum_j \sigma_{ij} \lambda_D\left(Q_j\right). \tag{11}$$

Capacity constraint is strong if $C'(P_i)$ is large, especially when it is larger than C_i. Standardizing $C'(P_i)$ by C_i, we define *capacity constraint measure of facility* P_i:

$$\lambda_C\left(P_i\right) = \frac{C'\left(P_i\right)}{C_i} = \frac{\sum_j \sigma_{ij} \lambda_D\left(Q_j\right)}{C_i}. \tag{12}$$

For facility P_i, distance constraint is strong if its users are highly restricted by the distance constraint. We thus define *distance constraint measure of facility* P_i as the average distance constraint measure of its users:

$$\lambda_D\left(P_i\right) = \frac{\sum_j \sigma_{ij} \lambda_D\left(Q_j\right)}{\sum_j \sigma_{ij}}. \tag{13}$$

Similarly, *capacity constraint measure of user* Q_i is the average capacity constraint measure of accessible facilities:

$$\lambda_C\left(Q_j\right)=\frac{\sum_i \sigma_{ij}\lambda_C\left(P_i\right)}{\sum_i \sigma_{ij}}$$

$$=\frac{\sum_i \sigma_{ij}\frac{\sum_j \sigma_{ij}\lambda_D\left(Q_j\right)}{C_i}}{\sum_i \sigma_{ij}} \cdot$$

(14)

The above four measures indicate the strength of constraints. They are zero or positive and show a large value when constraints are restrictive on facility location. Distance constraint measures ranges from zero to one while capacity constraint measures do not have any upper limit.

In addition to the measures, it is useful to consider facilities that definitely have to remain to meet the distance constraint. A facility that has a user who has only one accessible facility is called *indispensable facility*. Facilities whose distance constraint measure $\lambda_D(P_i)=1$ are indispensable facilities. Note, however, the measure of indispensable facilities is not always equal to one. The measure becomes one only if $\lambda_D(Q_j)=1$ for all the users of a facility.

2.4 Analysis of alternative plans

We propose three basic statistics to analyze the alternative plans in the following,

First measure is *adoption rate* of facility P_i denoted by a_i. It is the ratio of alternative plans in Ψ_F in which facility P_i remains:

$$a_i=\frac{1}{T}\sum_k s_{ki} \ .$$

(15)

Second measure is *capacity utilization rate* of facility P_i denoted by u_i. It is the average ratio of the number of facility users to the capacity of facility defined by

$$u_i=\frac{\sum_k s_{ki}\sum_j y'_{ijk}}{C_i\sum_k s_{ki}} ,$$

(16)

where y'_{ijk} is a binary function that indicate whether Q_i is allocated to P_i in plan S_k.

Third measure is *complementarity rate* of facilities P_i and P_j. If either P_i or P_j appears in every plan, they are complementary to each other. To evaluate the closeness of this relationship, we consider a random process of facility choice. Let us assume that facilities P_i and P_j appear in a plan independently with probabilities of a_i and a_i, respectively. The probability that either P_i or P_j appears in a plan is

$$p_{ij}=a_i\left(1-a_j\right)+\left(1-a_i\right)a_j \, .$$

(17)

Suppose that either P_i or P_j appears in T_C out of T plans in Ψ_F. The probability that the number of plans that contain either P_i or P_j is less than T_C is given by

$$c_{ij}=\sum_{t=1}^{T_C-1} {}_T C_t \left\{a_i\left(1-a_j\right)+\left(1-a_i\right)a_j\right\}^t \left\{a_i a_j+\left(1-a_i\right)\left(1-a_j\right)\right\}^{T-t} \, .$$

(18)

We define this probability as the complementarity rate of facilities P_i and P_j. This measure evaluates the strength of complementarity between P_i and P_j. It ranges from zero to one, and shows a large value if two facilities are highly complementary to each other. Since this measure is statistically defined, it can also be used for statistical test. Facilities P_i and P_j are complementary to each other at significant rate 1-c_{ij}.

3. Application

This section applies the proposed method to school location planning in Inage and Wakaba Wards in Chiba City, Japan. These wards are located in a suburb of Tokyo.

There were 16 and 20 public elementary schools in 2009 in these wards, respectively (Figure 1). With a rapid decrease in birth rate, however, pupils of elementary schools have been decreasing since 1981. School reduction has been being discussed to improve educational environment of schools and economic efficiency of school management.

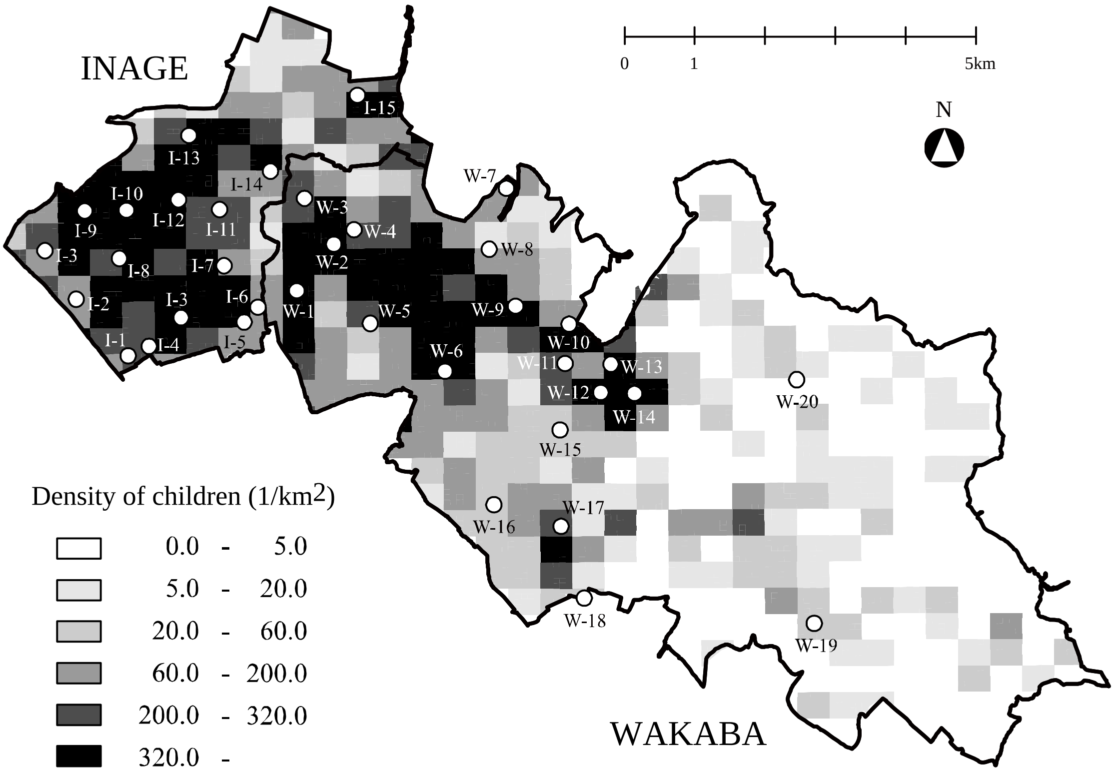

Figure 1 Public elementary schools and density distribution of children aged 6-12.

The Ministry of Education, Culture, Sports, Science and Technology provides that the maximum capacity of elementary school should not exceed 720 pupils and that the distance from home to elementary school should not exceed 4km. This distance, however, is too long in urban areas since pupils go to schools by walk. We thus set C_i=720 and $d_{\max}$=2km. School districts are determined by local administrations. Pupils in Inage Ward cannot go to schools in Wakaba Ward and vice versa.

Let us examine the present status of the distribution of elementary schools and that of distance constraint strength. Figure 2 shows that the distance constraint is strong where schools are sparsely distributed such as the east of Wakaba Ward. It is also restrictive near administrative boundaries, especially in the south of Wakaba Ward. However, as seen in the south of Inage Ward, the distance constraint is not effective if schools are densely located.

Indispensable schools are located primarily along administrative boundaries. As mentioned earlier, some of them show large $\lambda_D(P_i)$ values while others have small $\lambda_D(P_i)$ values. Schools B-5, B-6, and B-13 are indispensable though their $\lambda_D(P_i)$ is small.

Figure 3 shows the distribution of capacity constraint measure. This constraint is strong in the north of Inage Ward and the west of Wakaba Ward. As seen in Figure 1, the latter has many pupils than other area. Inage Ward, on the other hand, pupils are distributed more densely in the south rather than in the north. Weak capacity constraint in the south of Inage Ward suggests that more schools exist than those necessary for pupils in this area.

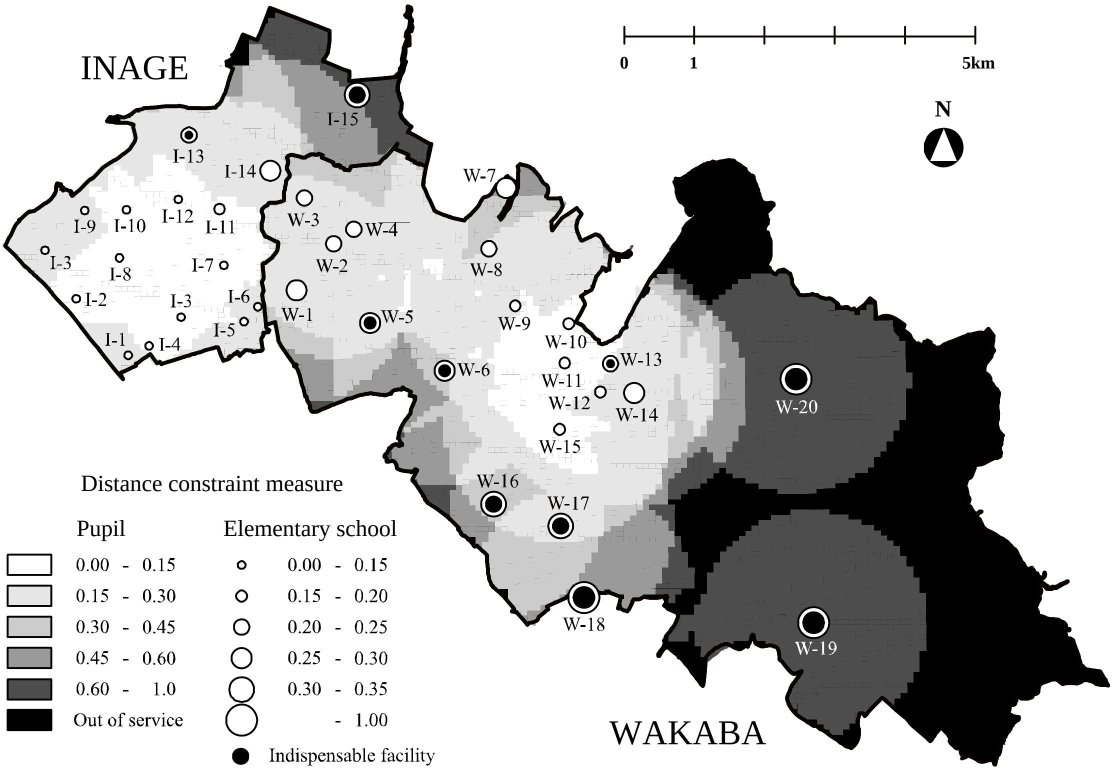

Figure 2 The distributions of distance constraint measures $\lambda_D(P_i)$ and $\lambda_D(Q_i)$. White circles containing black ones indicate indispensable schools.

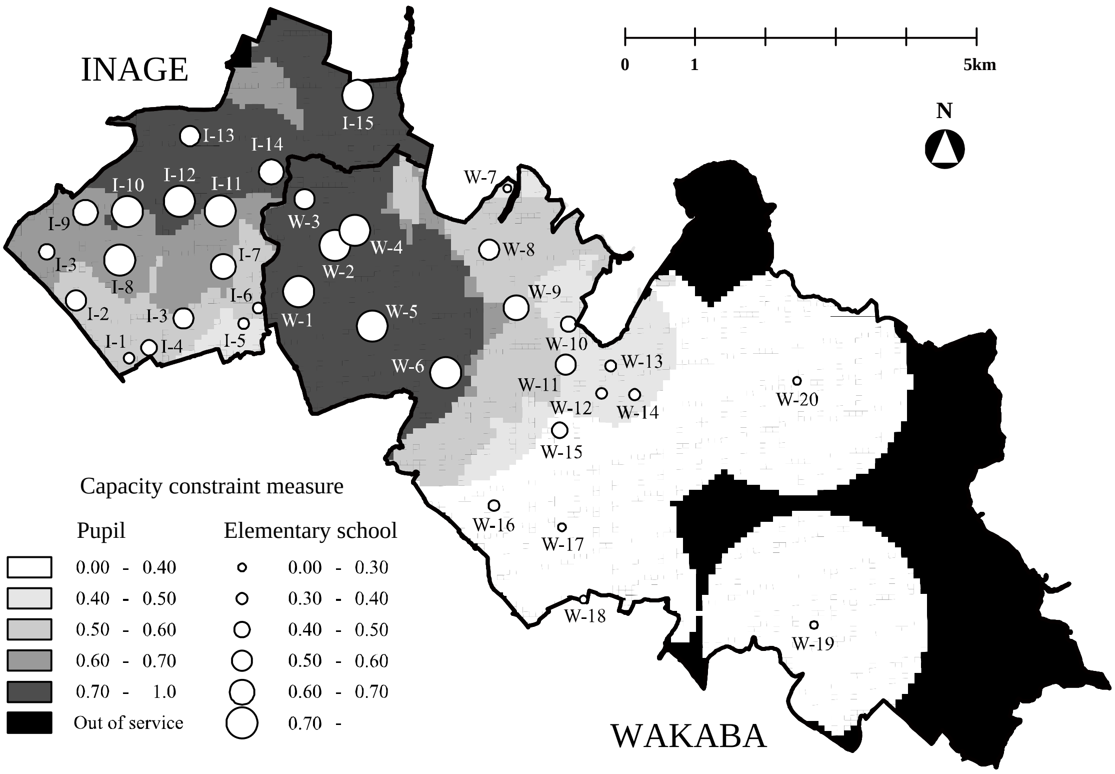

Figure 3 The distributions of capacity constraint measures $\lambda_C(P_i)$ and $\lambda_C(Q_i)$.

We then derive the minimum number of schools and alternative school plans. Under the constraints mentioned earlier elementary schools can be reduced from 16 and 20 to 11 and 15 in

Inage and Wakaba Wards, respectively. Using the method proposed in Section 2, we derived 100 alternatives in each ward.

Figure 4 shows the distribution of adoption rate a_i. In addition to nine indispensable schools, three schools show a_i=1.0; they are included in all the 100 plans. School W-7 shows a large $\lambda_D(P_i)$ in Figure 2 while schools I-14 and W-1 show large $\lambda_C(P_i)$ in Figure 3. This implies that school W-7 is included in all the plans in order to satisfy the distance constraint while I-13 and W-1 are adopted to meet the capacity constraint.

Let us look at schools of low adoption rate. They are located in the south of Inage Ward and in the center of Wakaba Ward. Figure 6 shows that schools I-1 and I-4, I-5 and I-6, W-9 and B-10, B-9 and B-11, B-11 and B-15 are complementary to each other. This implies that it is enough to keep either of two schools in these complementary pairs. Schools W-12 and W-14 are not complementary to any other school. Whether they are closed or not should be discussed in consideration of wider options.

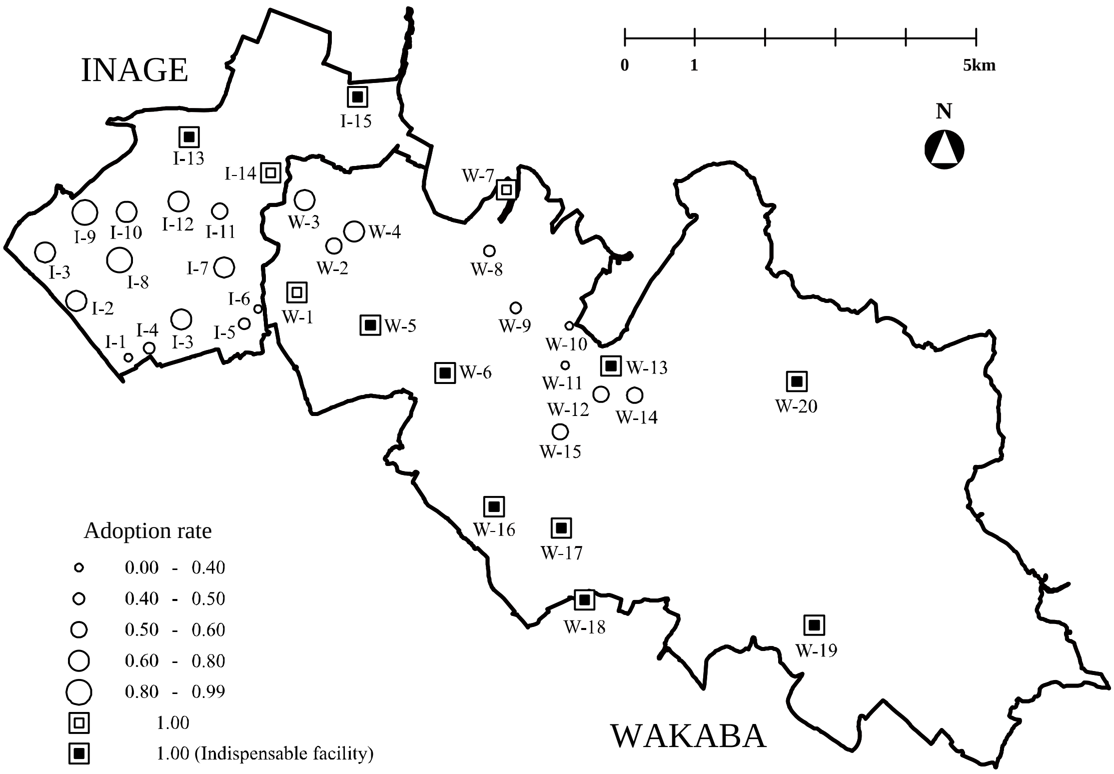

Figure 4 Adoption rate of elementary schools.

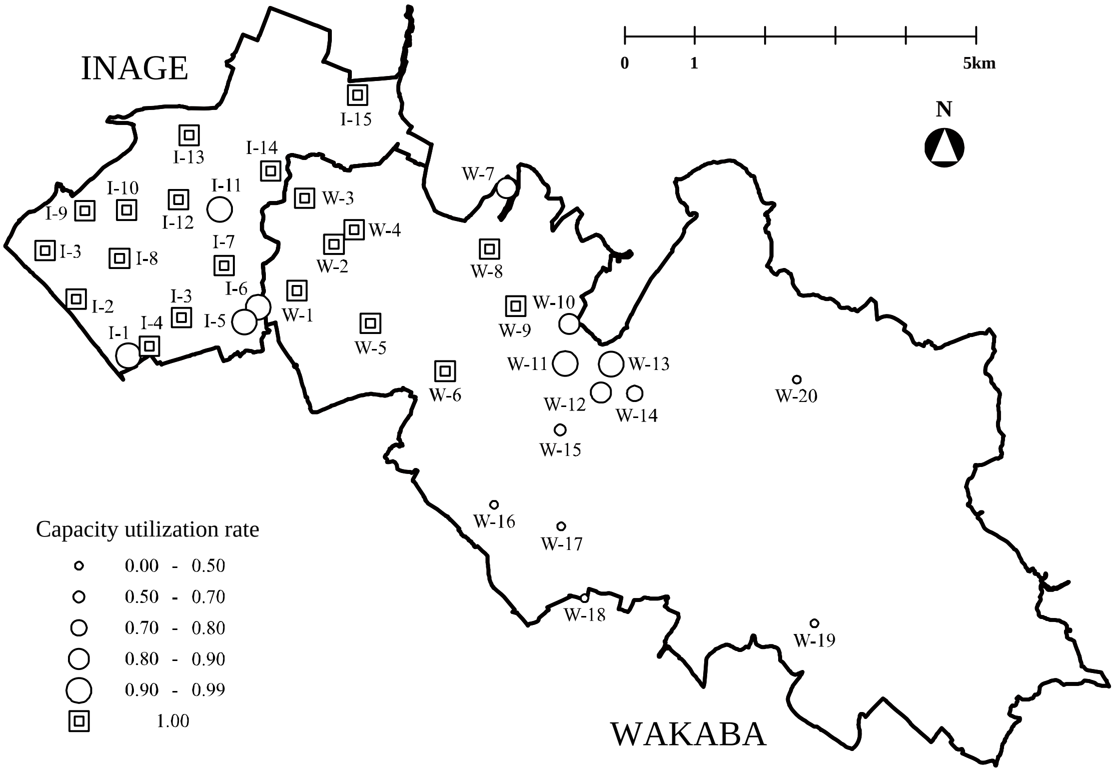

Figure 5 Capacity utilization rate of elementary schools.

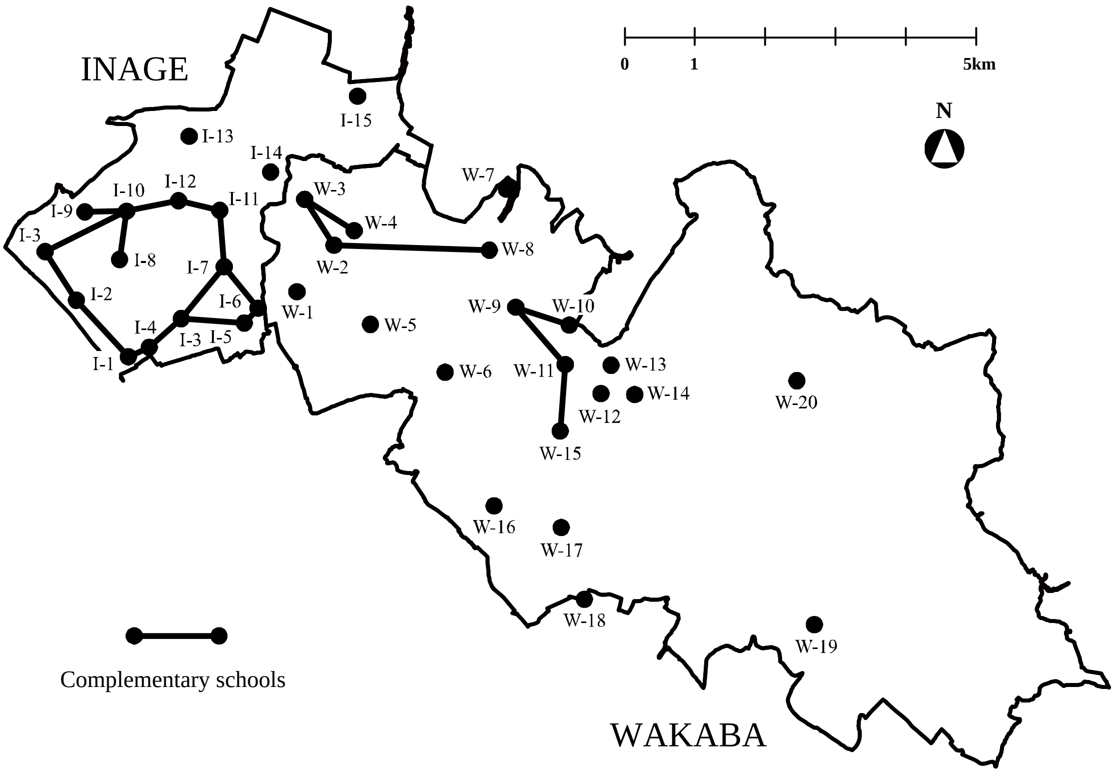

Figure 6 Pairs of complementary schools. Pairs of schools whose complementary rate is larger than 0.95 are connected by links.

Let us then move to a wider variety of policies other than school closure. The capacity and distance constraints are provided by the Ministry of Education, Culture, Sports, Science and Technology. In reality, however, they are not absolute rules to be satisfied. If we adopt them very strictly, it greatly reduces the economic efficiency of school management. For instance, the distance constraint in rural areas requires many schools of a few pupils. Therefore, it is realistic to consider the relaxation of constraints in light of efficient school management. Schools W-16, W-17, W-18, W-19, and W-20 show high $\lambda_D(P_i)$ in Figure 2 but low $\lambda_C(P_i)$ in Figure 3. This implies that efficiency of school management will be improved by relaxing the distance constraint. Introduction of school bus and bicycle attendance is worth discussing.

On the other hand, there are schools of high capacity and low distance constraints such as those in the middle of Inage Ward and the northwest of Wakaba Ward. In suburban areas school buildings and yards can be extended to relax the capacity constraint s. It is possible in schools such I-11, I-14, W-1, W-2, W-3, and W-4 to improve the efficiency of school management.

At present school attendance to neighboring wards is not permitted. If it is permitted pupils around schools W-1, W-2, and W-3 can go to schools of low capacity utilization rate such as I-5, I-6, and I-11. This policy is effective especially in urban-rural fringe where population density changes drastically.

4. Conclusion

This paper has proposed a new decision support method for facility location planning in population decrease. A focus is on urban facilities whose maximum capacity and service distance are given as constraints. Minimization of facilities is formulated as a mathematical optimization problem with capacity and distance constraints. Solutions of the problem are analyzed by using maps showing the spatial distribution of summary statistics. The method was applied to school location planning in Chiba City, Japan. The result provided useful empirical findings as well as illustrated its utilization in school location planning.

We finally discuss some limitations and extensions of the paper for future research. First, analysis heavily depends on the ability of analysts including politicians and administrative staffs. Unfortunately, they do not necessarily have much experience in quantitative analysis and interpretation of the result. To resolve this, implementation of the method in the computer environment should be considered. Automated system permits us to discuss a wider variety of plans. Implementation by using GIS and the Internet provides a visual and interactive environment for collaborative decision making. Second, a wider variety of plans should be considered within analytical process. In Section 3, we discussed the relaxation of constraints and school attendance across different administrative units. To make the discussion more objective it is desirable to evaluate such options quantitatively during mathematical process. Third, the method should be applied to urban facilities other than those of capacity and distance constraints. Distance constraint implicitly assumes that people directly go and go back from home to facilities. However, this assumption does not always hold in reality. Post offices and banks are often used on the way from home to office. People visit many shops in shopping district. The method should be extended to treat such kind of urban facilities.

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