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Decision support of spatiotemporal facility location: location planning of public facilities in population decrease

Yukio Sadahiro
Department of Urban Engineering, University of Tokyo

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Abstract

Population decrease is one of the most critical issues in urban planning in developed countries. This requires us to reconsider the location of public facilities for economic efficiency. This paper proposes a new decision support method of facility location in population decrease. The method consists of two phases: preparation and evaluation of alternative plans. Two methods are developed to generate draft plans, one utilizes a stochastic process of facility location while the other is based on spatial optimization of facility location. Quantitative measures are proposed to evaluate the draft plans. A focus is on the strength of constraints assumed in facility location. The measures are visualized in various forms including tables, figures, and maps. This helps us to understand the relationship between constraints and facility location, and consequently, leads to flexible planning of facility location. The method is applied to a location planning of elementary schools in Chiba City, Japan. It illustrates a concrete usage of the method proposed as well as provides empirical findings.

Keywords: *decision support, spatiotemporal facility location, public facilities, population decrease*

1. Introduction

Population decrease is one of the most critical issues in urban planning in developed countries (Farr, 2007; Langner and Endlicher, 2007). Local communities collapse and social capital decreases. Retail stores are unable to continue their business due to a sharp decrease in sales. Local governments are burdened with the cost of providing public services.

Location of public facilities has to be reconsidered to meet this new phase. Population decrease requires spatiotemporal planning of facility location with a long-term view.

What distinguishes public facilities from commercial ones is that the former have to serve all the residents at least at a minimum level of service. Hospitals, police stations, and fire stations are indispensable even in sparsely populated area. This usually conflicts with economical efficiency that is in a sense the most essential goal pursued by private companies. Though isolated facility location impedes efficient operation of facilities, it is still necessary if all the residents need that kind of facilities within a certain distance.

One method to solve the problem is visual analysis of the present status of facility location. Comparing the distributions of population and facilities, we would find the area where more facilities need to be built. Choropleth maps showing the density of population and facilities may be more appropriate for understanding the necessity of facilities at a local level.

Visual analysis is a powerful tool of solving spatial problems. Human eyes are powerful detectors of spatial patterns (Wood, 1992; MacEachren, 2004; Dodge *et al.*, 2008; Kraak and Ormeling, 2009). However, visual analysis is rather vague and often subjective whose result is not firm enough to persuasive all the participants of location planning.

Another option is spatial optimization technique that gives an optimal location of facilities. It mathematically calculates the location of facilities that is optimal in a certain aspect such as the operation and management cost of facilities, travelling cost of facility users, and so forth, under a minimum level of service (Mirchandani and Francis, 1990; Drezner, 1995; Drezner and Hamacher, 2004).

One drawback of spatial optimization is that it considers a highly abstract model of the real world. Homogeneity is assumed for both facilities and their users to a considerable degree; homogeneous distribution of users, simple behavior of facility choice, services of facilities, and so forth. Such assumptions are not easily acceptable in a practical sense, and consequently, facility location derived from spatial optimization often sounds unrealistic.

Location of public facilities involves public administrations, private companies, non-profit organizations and local communities. Collaborative planning is indispensable where facility location is discussed from a number of different perspectives to reach a desirable plan reasonable for many of participants (Healey, 1997; Saaty and Peniwati, 2007).

To support collaborate planning of public facilities, this paper proposes a new method of decision support of spatiotemporal facility location. An important key to success of such a collaborative planning is to understand the problem that they are facing with. It is essential to grasp correctly the present and possible future situations, the latter of which are represented as future plans. The difficulties, problems and advantages involved in these situations have to be evaluated and presented in a plain but objective way.

Using visualization and spatial optimization techniques, this paper shows a method of understanding the structure of the problem to be solved. This paper focuses on a location planning of one type of facilities during a certain period of time. There already exist facilities that serve the residents in the neighborhood. Due to population decrease, however, facilities should be reduced as much as possible for economic efficiency, which involves integration, conversion and closure of existing facilities. The number of facilities should be reduced as much as possible.

Discussion starts with static facility location. We then extend it to the spatiotemporal domain by taking into account the temporal dimension. Section 2 outlines the general setting of the method, followed by a discussion of evaluating a single plan of facility location. Section 3 proposes a method of preparing draft plans of facility location. Two methods are developed to generate draft plans, one utilizes a stochastic process of facility location while the other is based on spatial optimization of facility location. Section 4 evaluates a set of draft plans by quantitative measures. A focus is on the strength of constraints assumed in facility location. Section 5 extends the proposed method to the spatiotemporal domain. Section 6 applies the method to a location planning of elementary schools in Chiba City, Japan. Section 7 summarizes the conclusions with discussion.

2. Evaluation of a single plan

2.1 General setting

Suppose a two-dimensional region S in which facilities and their users are distributed. Let F_i (i=1, 2, ..., M) and U_j (j=1, 2, ..., N) be the ith existing facility and jth user of facilities, respectively. The capacity of facility i is denoted by c_i.

Distance between facility F_i and user U_j is represented by d_{ij}. For convenience it is converted into accessibility function. Examples include

$$a_{ij}(\Omega) = \begin{cases} 1 & \text{if } d_{ij} \le d_{\max} \\ 0 & \text{otherwise} \end{cases}, \tag{1}$$

where users can go to facilities within distance $d_{\max}$.

Let Ω be a plan of facility location in which whether each facility is kept open or closed is represented by a binary function:

$$f_i(\Omega) = \begin{cases} 1 & \text{if } F_i \text{ remains open in } \Omega \\ 0 & \text{otherwise} \end{cases}. \tag{2}$$

2.2 Evaluation of location plan

Given plan Ω, we consider a stochastic choice model of facility users. The model assumes that every user randomly chooses one facility from accessible ones. The number of facilities accessible from user U_j is

$$m_j(\Omega) = \sum_i a_{ij}(\Omega). \tag{3}$$

Consequently, the probability of user U_j choosing F_i is

$$p_{ij}(\Omega) = \frac{1}{m_j(\Omega)}. \tag{4}$$

The above model permits us to evaluate the necessity of each facility in two aspects, that is, accessibility of users and capacity of facilities. If a user has many accessible options, facility choice is flexible and thus the necessity of each facility is relatively low. This is represented by the *necessity of* F_i *by accessibility limitation* in plan Ω given by

$$\nu_i^A(\Omega) = \max_j p_{ij}(\Omega). \tag{5}$$

It is the highest probability of choosing F_i among its accessible users, ranging from zero to one. Facility F_i shows $\nu_i^A(\Omega)=1$ if it has a few users of only one accessible facility even though the others have many options. This typically happens in rural areas where facility users have fewer options. Lower boundary of $\nu_i^A(\Omega)$ is zero, which appears when many facilities are accessible to all the users.

The necessity of facilities also depends on their capacity compared to the number of their potential users. Facilities are necessary if they are expected to have many users. The *necessity of* F_i *by capacity limitation* in plan Ω is defined as

$$\nu_i^C(\Omega) = \min\left\{\frac{n_i(\Omega)}{c_i(\Omega)}, 1.0\right\}. \tag{6}$$

The first term is the ratio of the number of users to the capacity of facility F_i. The second term is added to limit the range of $\nu_i^C(\Omega)$ from zero to one, in order to make $\nu_i^C(\Omega)$ comparable to $\nu_i^A(\Omega)$. This implies that F_i is considered indispensable if F_i has more users than its capacity.

The overall necessity of a facility depends on the above two necessities. We define the *absolute necessity of* F_i in plan Ω:

$$\nu_i(\Omega) = \max\left\{\nu_i^A(\Omega), \nu_i^C(\Omega)\right\}. \tag{7}$$

A large $\nu_i(\Omega)$ implies that F_i is quite necessary for users in its neighborhood. On the other hand, if $\nu_i(\Omega)$ is very small, F_i is not necessary so that it should be closed for economic efficiency in plan Ω.

3. Preparation of draft plans

As mentioned earlier, collaborative planning often starts with the discussion on draft plans prepared in advance. Though new plans are often proposed in discussion, draft plans are very useful because they give a concrete view of facility location.

3.1 Stochastic method

A simple method is based the evaluation of the present location of facilities mentioned above. Let Ω_0 be the present location of existing facilities. The necessity of each facility at present is given by $\nu_i(\Omega_0)$.

We consider a stochastic process of facility location where the probability of a facility being kept open is given by its necessity. This process is a multinomial distribution where the probability of facility F_i remains open is given by $\nu_i(\Omega_0)$. We can obtain any number of draft plans by determining $f_i(\Omega_k)$ according to the multinomial distribution.

3.2 Optimization method

A more sophisticated method is to utilize the spatial optimization technique that gives an optimal location of facilities (Mirchandani and Francis, 1990; Daskin, 1995; Drezner, 1995; Drezner and Hamacher, 2004). Given the setting mentioned above, we consider the reduction of facilities appropriate for population decrease.

The objective function is the number of facilities. Every user is assigned one accessible facility under the condition that the number of users assigned to a facility does not exceed its capacity. This optimization problem is represented as

Problem $\mathbf{P_1}$:

$$\min_{f_i(\Omega), x_{ij}(\Omega)} \sum_i f_i(\Omega), \tag{8}$$

subject to

$$\begin{aligned} \sum_j x_{ij}(\Omega) &\leq c_i, \forall j \\ x_{ij}(\Omega) &\leq f_i(\Omega), \forall i, j \\ x_{ij} a_{ij}(\Omega) &= 1, \forall i, j \\ \sum_j x_{ij}(\Omega) &= 1, \forall j \end{aligned},$$

(9)

where $x_{ij}(\Omega)$ is a binary function representing the assignment of user U_j to facility F_i (similar discussion can be found in Sadahiro & Sadahiro, 2009). The third constraint includes $a_{ij}(\Omega)$, a binary function that represent the accessibility of facility F_i from user U_j in plan Ω. Examples include

$$a_{ij}(\Omega) = \begin{cases} 1 & \text{if } d_{ij} \le d_{\max} \\ 0 & \text{otherwise} \end{cases},$$

(10)

where users can go to facilities within distance $d_{\max}$.

Solving the above problem, we obtain the minimum number of facilities $M_{\min}$ and a set of locations that minimize the number of facilities. However, not only a single set of locations gives $M_{\min}$. One method to derive other sets of locations is to change the value of binary variable f_i from zero to one for one facility and one to zero for the other one. Examining whether the location set satisfies all the given constraints, we can verify whether or not the set is another solution. We repeat this process until we obtain desirable number of alternative plans (for details, see Sadahiro and Sadahiro, 2009).

4. Evaluation of draft plans

4.1 Basic measures

Section 2 describes a method of evaluating a single plan of location facilities. The method is also effective to evaluate individual facilities in draft plans.

In addition to necessity measures, basic descriptive measures such as the expected number of users and the average distance from home to school are useful to grasp the properties of draft plans. They are defined as

$$n_i(\Omega) = \sum_j p_{ij}(\Omega).$$

(11)

and

$$n_i(\Omega) = \sum_j p_{ij}(\Omega) d_{ij} ,$$

(12)

respectively.

4.2 Measurement of constraint strength: 1. Evaluation of draft plans

A key to success in location planning is to understand the properties of constraints, say, how and where they limit available options, how they can be relaxed, and so forth. This permits us to consider a wider variety of political options in location planning such as the expansion of existing facilities and improvement of accessibility.

To this end, we consider the flexibility in location planning with respect to its constraints. Let Λ and #(Λ) be a set of alternatives and its number of elements, respectively. Each alternative in draft set Λ is denoted by Ω_k. We consider a stochastic process to describe the choice of a final plan from draft alternatives. The probability of plan Ω being chosen from Λ is denoted by $P(\Omega)$. The *flexibility* of draft set Λ is measured by the entropy of this process:

$$\Phi(\Lambda) = -\sum_k P(\Omega_k) \log P(\Omega_k) . \tag{13}$$

If all the alternatives are equally plausible or no information is available on the feasibility of individual plans, the flexibility is maximized:

$$\Phi(\Lambda) = \log \#(\Lambda) . \tag{14}$$

The flexibility becomes zero when only one draft plan is feasible.

Given M facilities, in theory, we have 2^M possible combinations of facility location denoted by Λ_0. If they are equally feasible, the flexibility is

$$\Phi(\Lambda_0) = M \log 2 . \tag{15}$$

This, however, decreases because constraints limit the alternatives. Consequently, the strength of a constraint can be measured by a decrease in flexibility:

$$\Delta\Phi(\Lambda, \Lambda_0) = \Phi(\Lambda_0) - \Phi(\Lambda) . \tag{16}$$

A large value indicates that a constraint is highly restrictive on location planning. If all the alternatives are equally feasible in both Λ and Λ', the strength of constraint is given by

$$\Delta\Phi(\Lambda', \Lambda) = \log \#(\Lambda) - \log \#(\Lambda') . \tag{17}$$

If draft plans are derived by spatial optimization, the minimum number of facilities $M_{\min}$, is obtained. Let Λ_M be the set of all the combinations giving $M_{\min}$. Its number of elements is then given by

$$\begin{aligned} \#(\Lambda_M) &= {}_M C_{M_{\min}} \\ &= \frac{M!(M - M_{\min})!}{M_{\min}!} . \end{aligned}$$

(18)

Comparing Λ_M with Λ_0, we can evaluate how the demand for economic efficiency reduces the flexibility of facility location. We call this *efficiency constraint* evaluated by

$$\Delta\Phi\left(\Lambda_M,\Lambda_0\right) = M\log 2 - \log\#\left(\Lambda_M\right)$$
$$= M\log 2 - \sum_{i=M_{\min}+1}^{M}\log i - \sum_{i=1}^{M-M_{\min}}\log i \,.$$

(19)

In our setting, accessibility and capacity are two major constraints of facility location. To measure their effect on facility location, we prepare three alternative sets Λ_A, Λ_C, and Λ_M. Sets Λ_A and Λ_C satisfy only the accessibility and capacity constraints, respectively, to achieve the minimum number of facilities $M_{\min}$.

The strength of *accessibility constraint* is measured by

$$\Delta\Phi\left(\Lambda_A,\Lambda_M\right) = \Phi\left(\Lambda_M\right) - \Phi\left(\Lambda_A\right).$$

(20)

Similarly, the strength of *capacity constraint* is

$$\Delta\Phi\left(\Lambda_C,\Lambda_M\right) = \Phi\left(\Lambda_M\right) - \Phi\left(\Lambda_C\right).$$

(21)

To evaluate draft plans generated by the stochastic method proposed in Section 3.1, we can use the necessity measure as the probability of facilities being chosen in the final plan. Let Λ_S be a set of draft plans generated by the stochastic method. Since the final plan follows the multinomial distribution where the probability of facility F_i remains open is $\nu_i(\Omega_0)$, the flexibility is given by

$$\Phi\left(\Lambda_S\right) = -\sum_i \left\{\nu_i\left(\Omega_0\right)\log\nu_i\left(\Omega_0\right) + \left(1-\nu_i\left(\Omega_0\right)\right)\log\left(1-\nu_i\left(\Omega_0\right)\right)\right\}.$$

(22)

4.3 Measurement of constraint strength: 2. Evaluation of individual facilities

The strength of constraints can also be evaluated for each facility. Let p_i be the probability that F_i is chosen to be kept open in the final plan from Λ. The flexibility of whether F_i is kept open or closed is

$$\phi\left(F_i;\Lambda\right) = p_i\log p_i + \left(1-p_i\right)\log\left(1-p_i\right).$$

(23)

It the probability changes from p_i to $p_{i'}$' by adding a new constraint to obtain draft set Λ', its strength is given by

$$\Delta\phi\left(F_i;\Lambda',\Lambda\right)=p_i\log p_i+\left(1-p_i\right)\log\left(1-p_i\right)-\left\{p_i'\log p_i'+\left(1-p_i'\right)\log\left(1-p_i'\right)\right\}.$$

(24)

We should note that constraints do not always increase the necessity of facilities. The flexibility can either increase or decrease by constraint, and consequently, the strength of a constraint can be either positive (lower the flexibility) or negative (heighten the flexibility).

However, to discuss the closure of existing facilities, it is useful and intuitive to focus on the negative effect of constraints because they usually increase the necessity of existing facilities. In such a case, strength of constraint should be evaluated by

$$\Delta\varphi\left(F_i;\Lambda',\Lambda\right)=p_i'\log p_i'-p_i\log p_i.$$

(25)

If draft set Λ is generated by the stochastic method, the above equation becomes

$$\Delta\varphi\left(F_i;\Lambda_S,\Lambda_0\right)=\nu_i'\left(\Omega_0\right)\log\nu_i'\left(\Omega_0\right)-\nu_i\left(\Omega_0\right)\log\nu_i\left(\Omega_0\right).$$

(26)

Let us consider the effect of accessibility and capacity constraints on individual facilities. Let p_i^A and p_i^C be the probabilities that F_i is chosen to be kept open in the final plan from Λ_A and Λ_C, respectively. The strength of accessibility constraint is given by

$$\Delta\varphi^A\left(F_i;\Lambda_A,\Lambda\right)=p_i^A\log p_i^A-p_i\log p_i.$$

(27)

We call this *demand for* F_i *by accessibility constraint*. We can similarly define *demand for* F_i *by capacity constraint*:

$$\Delta\varphi^C\left(F_i;\Lambda_C,\Lambda\right)=p_i^C\log p_i^C-p_i\log p_i.$$

(28)

The measures proposed above are represented as tables, figures and maps. They are served for understanding not only the properties of alternative plans but also the present status of facility location. Let us suppose, for instance, the case where many facilities show small $\Delta\varphi^C(F_i;\Lambda_C,\Lambda)$ but large $\Delta\varphi^A(F_i;\Lambda_A,\Lambda)$. The former implies that the users of facilities are fewer than their capacity, while the latter indicates that the facilities are still necessary due to the lack of accessibility. In such a case, improvement of accessibility increases the flexibility of facility location, which leads to the closure of inefficient facilities. On the other hand, if facility F_j shows a large $\Delta\varphi^C(F_i;\Lambda_C,\Lambda)$ and a small $\Delta\varphi^A(F_i;\Lambda_A,\Lambda)$, F_j is full of its users in its neighborhood. Expansion of facilities around F_i would be effective to decrease the necessity of F_i, and consequently, increases the flexibility of facility location.

5. Spatiotemporal facility location

This section extends the method proposed above to the spatiotemporal domain.

5.1 General setting

Let T={T_0, T_1} be a closed time-period. Facility location is discussed in the product $ST=S\times T$. The distribution of facility users changes over time because of birth, death and social fluctuation. It is thus represented as a set of binary functions of variable t. User U_i is represented as a binary function

$$u_j(t)=\begin{cases}1 & \text{if } U_j \text{ exists at time } t\\ 0 & \text{otherwise}\end{cases}. \tag{29}$$

Facility F_i in plan Ω is also represented by a binary function

$$f_i(t;\Omega)=\begin{cases}1 & \text{if } F_i \text{ exists at } t \text{ in } \Omega\\ 0 & \text{otherwise}\end{cases}, \tag{30}$$

The capacity of F_i at time t is denoted by $c_i(t)$. The accessibility of facility F_i from user U_j is $a_{ij}(t;\Omega)$.

5.2 Evaluation of a single plan

The measures proposed in Section 2.2 are calculated at any time during {T_0, T_1}. Their spatiotemporal equivalent is their integration with respect to t from T_0 to T_1. Standardized measures may be more convenient obtained by dividing them by time length T_1-T_0.

5.3 Preparation of draft plans

Spatiotemporal planning is an extension of spatial planning to the temporal domain. A facility location is represented as a zero-dimensional object in the spatial dimension, but a one-dimensional object in the temporal dimension. This causes much difficulty in spatiotemporal planning.

A method to solve this problem is to approximate a continuous temporal space by a discrete space. We choose sample points at a regular interval Δt during {T_0, T_1}, propose draft plans at each sample point, and interpolate them on the temporal dimension.

Spatial optimization can be utilized in several ways. A simple extension is to optimize the facility location at every sample point in T, that is,

Problem $\mathbf{P_2}$:

$$\min_{f_i(t;\Omega),x_{ij}(t;\Omega)} \sum_i \sum_k f_i\left(T_0+k\Delta t;\Omega\right), \tag{31}$$

subject to

$$\begin{aligned}
\sum_j x_{ij}\left(t;\Omega\right) &\le c_i\left(t;\Omega\right), \forall j \\
x_{ij}\left(t;\Omega\right) &\le f_j\left(t;\Omega\right), \forall i, j \\
x_{ij}a_{ij}\left(t;\Omega\right) &= 1, \forall i, j \\
\sum_i x_{ij}\left(t;\Omega\right) &= 1, \forall j
\end{aligned}.$$

(32)

The minimum number of facilities at time t is denoted by $M_{\min}(t)$. Unlike P_1, P_2 minimizes the total number of facilities during time period T. Problem P_2 permits flexible opening and closure of facilities. If a facility is designed for general purposes, it is easy to use it temporally for other purposes. Temporary closure of a short term is acceptable while it is often economically inefficient if it lasts so long.

In general, however, it is not always easy to use existing facilities for different purposes. This adds another constraint to problem P_2:

$$f_j\left(t';\Omega\right) \le f_j\left(t;\Omega\right), \forall j, \forall t \le t'.$$

(33)

We call this the *continuity constraint*. This new problem is denoted by P_3. The total number of facilities during T is generally larger than that derived by solving P_2.

Another method to handle with this problem is to solve P_2 at sample points in T and choose combinations of individual solutions that satisfy the above constraint. We call this problem P_4. Though this does not always minimize the total number of facilities during T, it is quite easy to solve P_4 than P_3 and it yields reasonable solutions close to the optimal one.

5.4 Evaluation of draft plans

Measures proposed in Section 4 are also useful to evaluate draft plans in the spatiotemporal domain. They are calculated at sample points and integrated with respect to t from T_0 to T_1.

In addition, it is useful to evaluate the strength of continuity constraint in spatiotemporal planning. In problem P_3, this additional constraint decreases the flexibility of facility location. Consequently, we can evaluate its effect by comparing the flexibility of the solutions of problems P_2 and P_3.

Suppose, for instance, two sets of alternative plans Λ_1 and Λ_2. The former follows the continuity constraint while the latter is free from it. Such alternatives are often obtained by spatial optimization discussed earlier. The difference of their flexibility indicates the effect of continuity constraint:

$$\Delta\Phi\left(\Lambda_1,\Lambda_2\right)=\Phi\left(\Lambda_2\right)-\Phi\left(\Lambda_1\right).$$

(34)

This measure can also be calculated for individual facilities. Suppose another set of alternatives Λ_3 where only facility F_i is permitted temporary closure. Then

$$\Delta\Phi\left(\Lambda_3,\Lambda_2\right)=\Phi\left(\Lambda_2\right)-\Phi\left(\Lambda_3\right)$$

(35)

indicates the effect of continuity constraint on F_i.

Another important aspect that should be considered in spatiotemporal planning is the uncertainty in future situation. The above discussion is based on the population projection that is inevitably uncertain to some extent. It is not problematic if temporary closure is permitted. If not, it is desirable to take this uncertainty into account in facility location.

One safe option is to close only facilities that rarely appear in alternative plans. Assuming that set Λ_1 is very plausible though it is based on uncertain population projection. Let Λ_4 be a subset of Λ_1 in which facility F_i does not appear in future. If we close F_i at present, the flexibility of facility location decreases from $\Phi(\Lambda_1)$ to $\Phi(\Lambda_4)$. Consequently, it would be better to close F_i if its *closure constraint* measured by

$$\Delta\Phi\left(\Lambda_4,\Lambda_2\right)=\Phi\left(\Lambda_2\right)-\Phi\left(\Lambda_4\right)$$

(36)

is relatively small.

Instead of choosing facilities to be closed, it may also be convenient to first choose facilities that should be kept open. This can be done by choosing F_i whose closure constraint is very high. Closing such facilities greatly limits the possible options in future.

Constraint strength on individual facilities can be evaluated in two different ways. One method is to consider the probability that F_i is chosen to be kept open at any time in the final plan from Λ. Another method is to evaluate the strength of constraints at every discrete time in T and take their average. In either case, the strength of constraints are measured by the method proposed in Section 4.3.

6. Empirical study

This section applies the proposed method to school location planning in Inage Ward in Chiba City, Japan. Inage ward is located 30 kilometers away in an eastern suburb of Tokyo. There is a railway station in the south around which shopping malls are clustered. They are surrounded by densely inhabited urban area. The north is a suburban area mostly covered with residential districts.

Inage ward has 16 public elementary schools and 6984 pupils in 2010 (Figure 1). With a rapid decrease in birth rate, however, pupils of elementary schools have been decreasing since 1981. Pupils are expected to decrease to 5224 and 4022 in 2030 and 2050, respectively. In 2050, every school has only 250 pupils, which is too small compared with the average size of elementary schools in Japan, School reduction is indispensable to keep educational environment of schools and economic efficiency of educational finance.

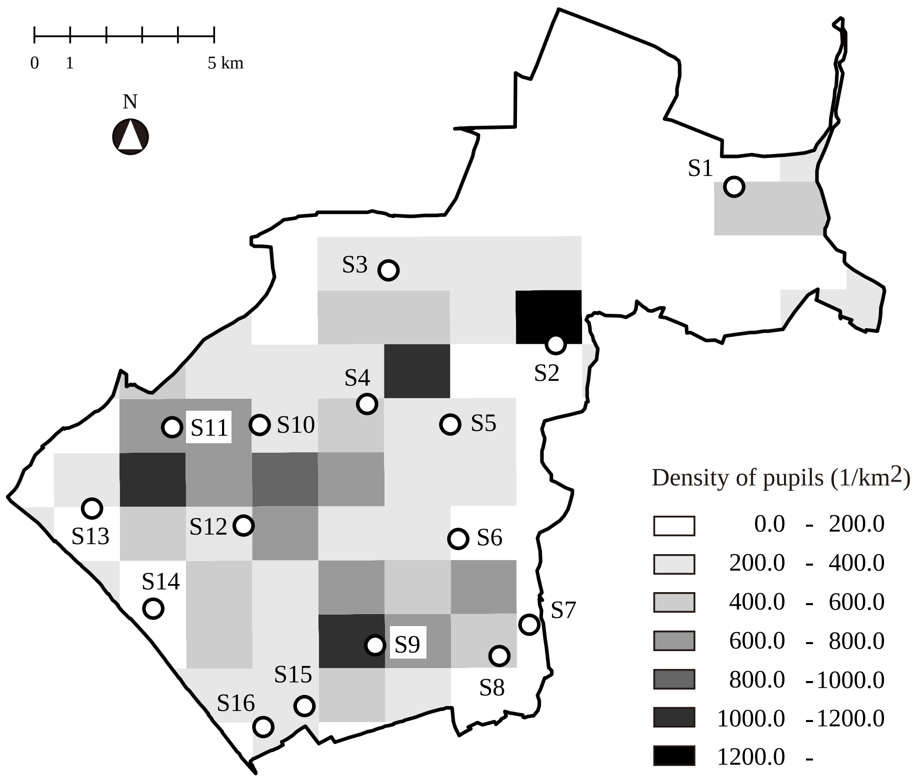

Figure 1 Public elementary schools and the density distribution of children aged 6-12.

We start with the examination of the present status of elementary schools. To this end, we take the existing 16 schools as the initial draft. As mentioned earlier, we can derive alternative plans from an initial plan by the method proposed in collaborative planning. Remember, however, evaluation is possible without actual derivation of alternative plans. Measures proposed in Section 4 can be calculated only from the location of facilities and facility users. Since our aim is to understand and evaluate the present status of elementary schools, we omit the derivation of alternative plans. For the capacity of schools and the accessibility of pupils, we adopt c_i=720 and d_{max}=2km in this paper following the standards provided by The Ministry of Education, Culture, Sports, Science and Technology. Pupils usually go to school by walk. School bus system is not popular in Japan.

We evaluate the present location of schools with respect to the distribution of pupils in 2010, 2030 and 2050. The expected number of pupils and their average distance from home to school were calculated for each school. Figures 2 and 3 shows the distributions of these measures and pupils in 2050.

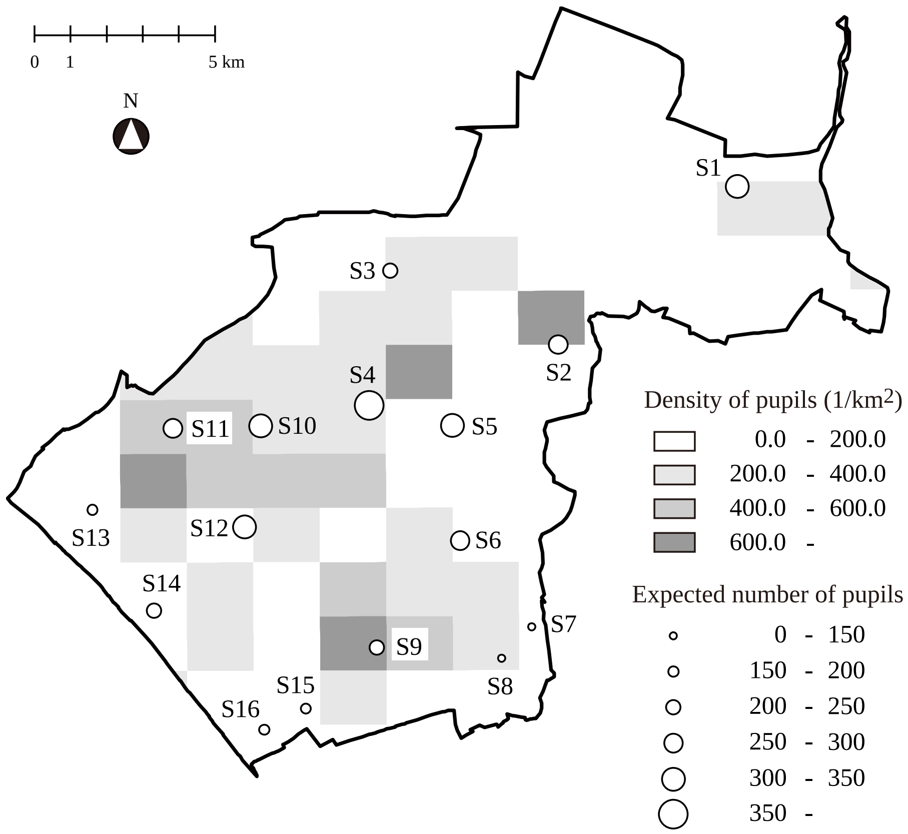

Figure 2 Expected number of pupils in 2050. Gray shades indicate the density distribution of children aged 6-12 in 2050.

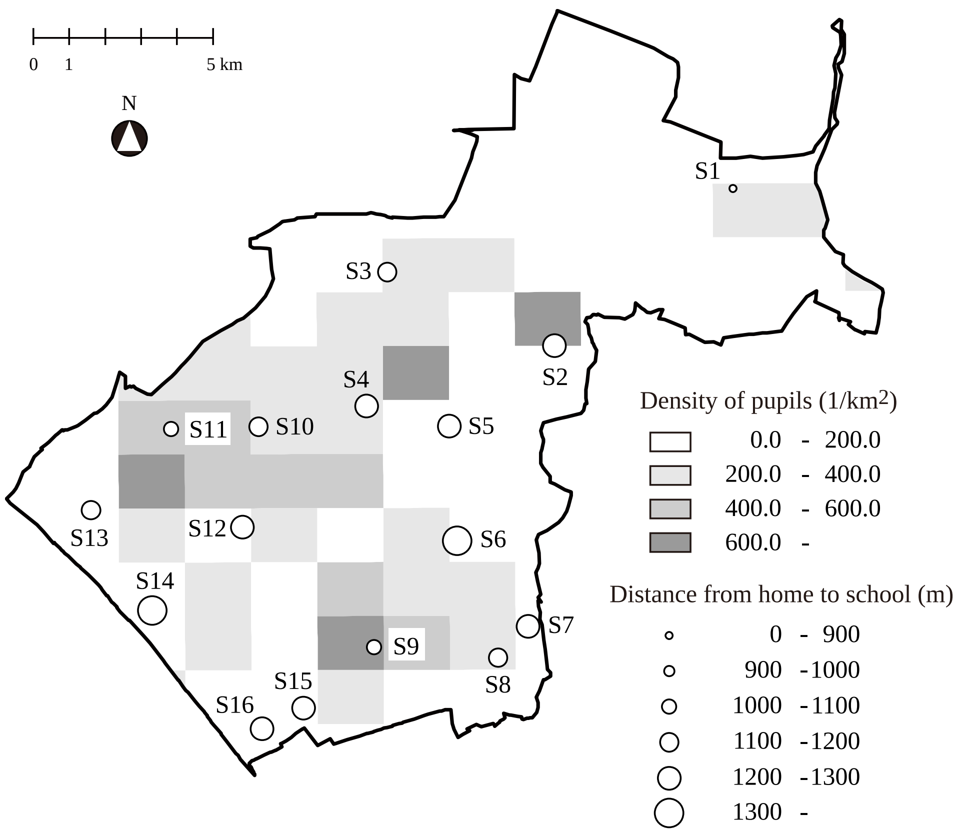

Figure 3 Average distance from home to school in 2050. Gray shades indicate the density distribution of children aged 6-12 in 2050.

As seen in Figure 2, all the schools have fewer pupils than their capacity in 2050. In urban area in the center of Inage ward, schools with many pupils in their close neighborhood such as S4, S5, S10, and S12 have many pupils. Schools S1 and S2, though located in the suburban area of Inage ward, also have many pupils because of its few competitors in its neighborhood. Others have fewer pupils so that they should be closed for economic efficiency. Overall impression is intuitively reasonable.

Unlike Figure 2, Figure 3 may seem counterintuitive. The average distance from home to school is not always shorter in urban area. This is because the school choice model assumes a uniform distribution with respect to all the accessible schools. Since it is independent of the distance to schools, it heavily depends on the spatial distribution of pupils and schools. School S1 shows a small value because its pupils are relatively clustered in its close neighborhood. Though schools in urban area also have pupils in their neighborhood, they also have many pupils in distant area. Schools S6 and S14 draw more pupils from distant area than their neighborhood, which increases the average distance from home to school.

Figure 4 shows the distribution of absolute necessity of schools in 2050. As seen in the figure, schools in the suburban area of Inage are highly necessary while those in urban area are less important.

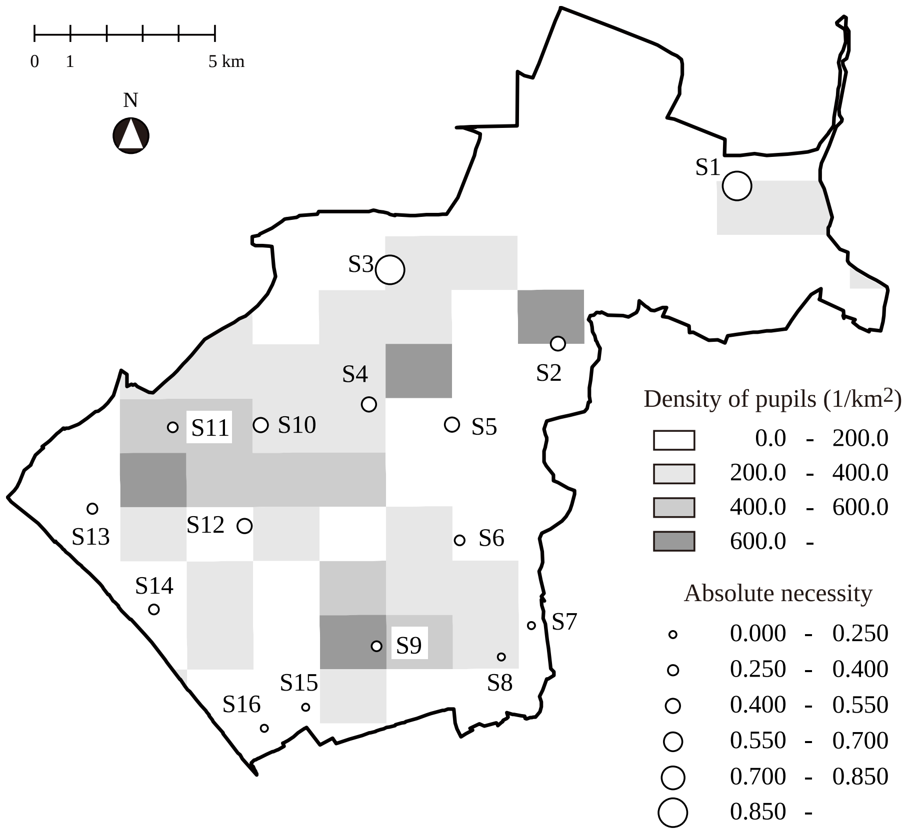

Figure 4 Absolute necessity of schools in 2050. Gray shades indicate the density distribution of children aged 6-12 in 2050.

To understand the causes of the necessity of schools, we then look at the demand for facilities by accessibility and capacity constraints in 2050. The former is shown in Figure 5, which is not necessarily correlated with Figure 3. As mentioned earlier, this measure becomes large if a school has pupils of few accessible options, which is typically observed in rural area. Schools S1, S2, and S3 shows a large value because they have a few pupils distantly located from schools. Schools in urban area have relatively small values.

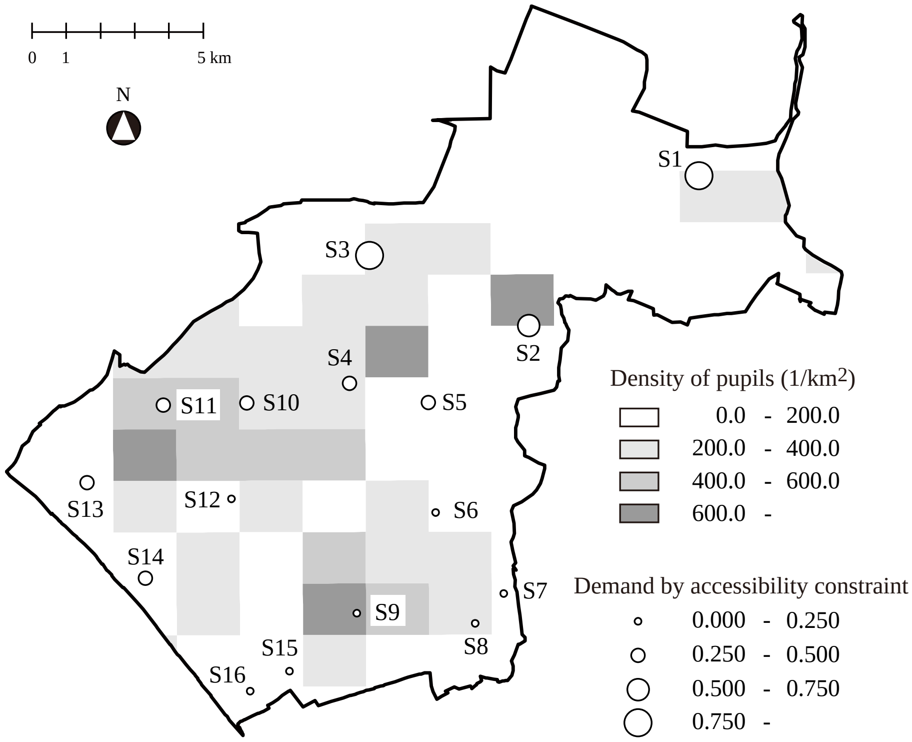

Figure 5 Demand by accessibility constraint in 2050. Gray shades indicate the density distribution of children aged 6-12 in 2050.

Figure 6 shows the demand by capacity constraint in 2050. It is very similar to Figure 2, which is quite reasonable. This measure indicates the ratio of the number of pupils to the capacity of schools. It is large in urban area, especially in the central area of Inage ward where schools are sparsely distributed among relatively many pupils. Schools in suburban area such as S1, S2, and S3 also show large values because they have many pupils in their neighborhood. Too many schools are located in urban area in the south of Inage ward. This happens because the pupils are expected to decrease rapidly until 2050.

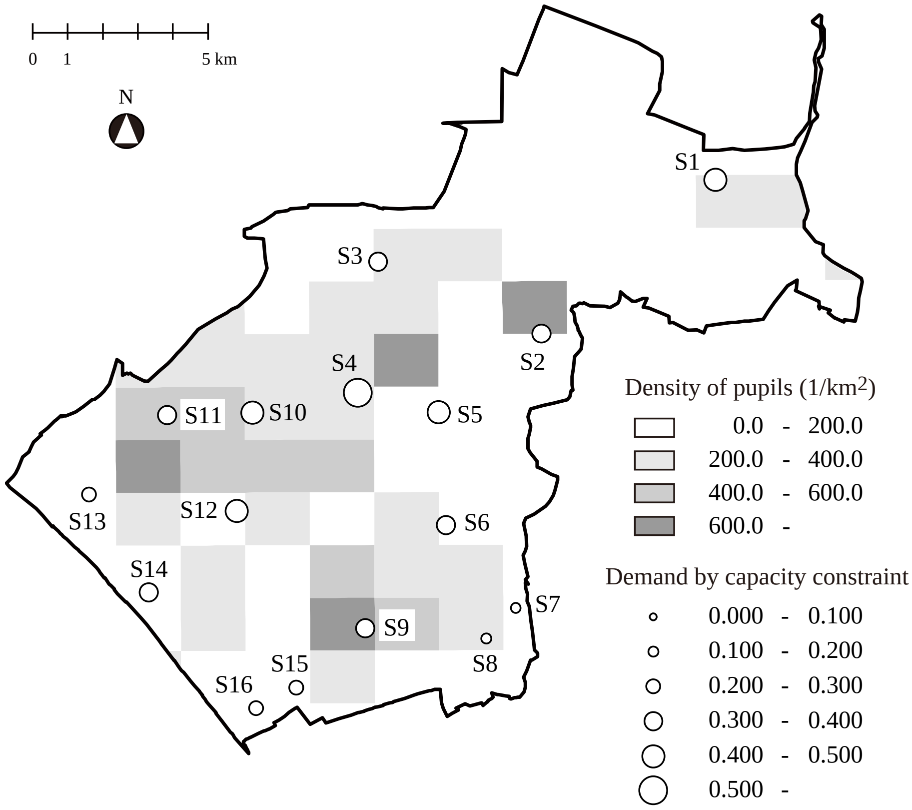

Figure 6 Demand by capacity constraint in 2050. Gray shades indicate the density distribution of children aged 6-12 in 2050.

Figure 7 shows the relationship between the capacity of schools and the flexibility of location planning. In general, larger schools can serve for more pupils, and consequently, increase the flexibility of choosing schools to be closed. This relationship is clearer in 2010 than in 2050 because there are more pupils in 2010; expansion of existing schools is effective in 2010 to consider a wider variety of options.

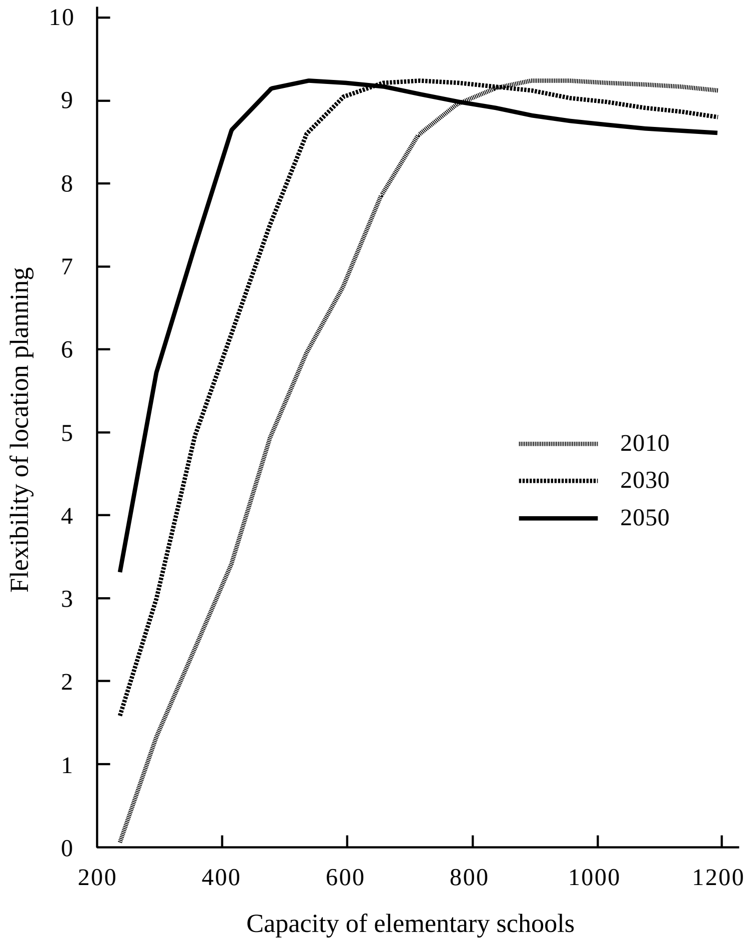

Figure 7 Relationship between the capacity of elementary schools and the flexibility of location planning, from 2010 to 2050.

As c_i increases, the flexibility increases rapidly and then gradually decreases. A local maximum appears in any case because the flexibility decreases when schools are too large compared with the number of pupils. Inefficient schools have small necessity measures so that they are likely to be closed. This decreases the total flexibility of school location, that is, only a few schools should be kept open while many others have to be closed.

Figure 8 shows the relationship between d_{max}, the maximum distance from home to school and the flexibility of location planning. As d_{max} increases, pupils have more options, and consequently, facility location becomes more flexible. The entropy is higher in 2030 than in 2010 and 2050 because of the same reason mentioned above. In general, the flexibility increases with a decrease of pupils. However, if there are too many schools compared with the number of pupils, whether inefficient schools are closed becomes more deterministic. This reduces the flexibility of location planning.

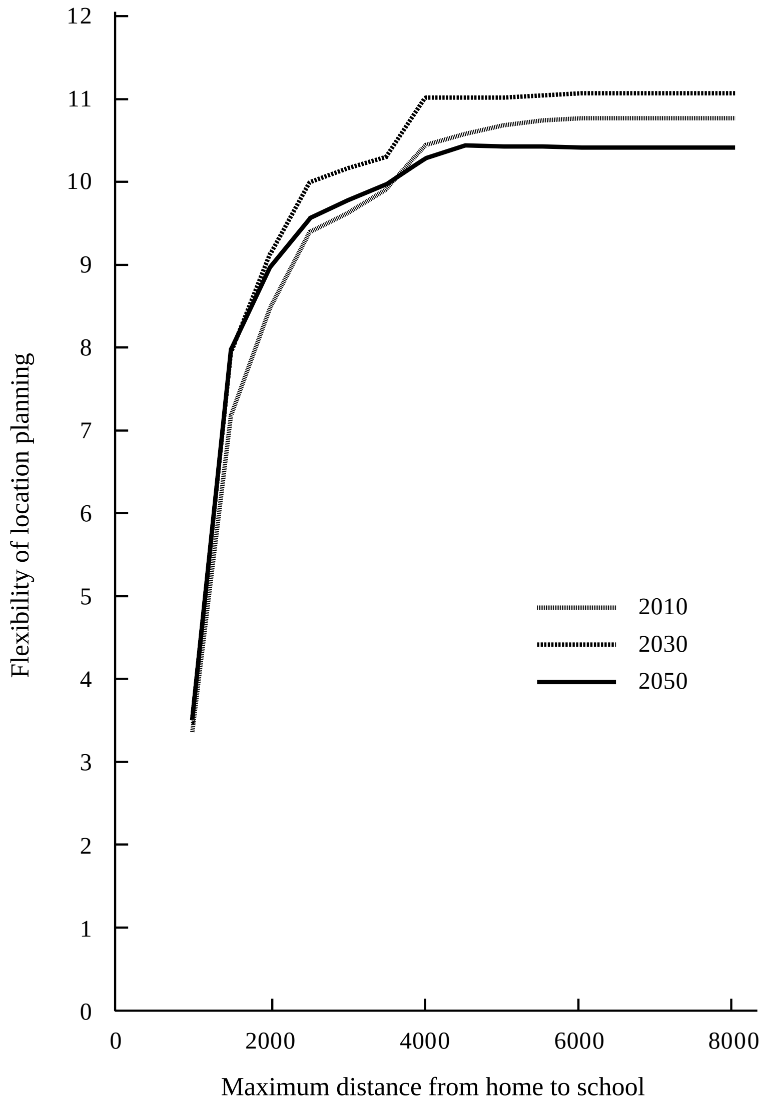

Figure 8 Relationship between the maximum distance from home to school and the flexibility of location planning, from 2010 to 2050.

As seen above, there are too many schools in Inage ward even in 2010 compared with the number of pupils. Without accessibility constraint we can reduce the schools from 16 to 10 (=6984/720) in 2010 and 6 in 2050. We thus adopt spatial optimization technique to derive more efficient location of elementary schools.

Spatiotemporal location optimization is formulated as problem P_2 shown in Section 5.3. Population forecast of pupils is available every five year from 2010 to 2050, among which 2010, 2030 and 2050 are chosen. A heuristic approach yielded the minimum number of schools denoted by $M_{min}(t)$ for time t. Three optimization problems are solved independently to yield the minimum number of schools 11, 9, and 7 in 2010, 2030, and 2050, respectively. Average number of pupils is 635, 580 and 575, which is around 80% of schools' capacity.

Alternative plans are then derived as much as possible that minimize the number of schools under the given constraints (for details, see Iwamoto, 2010). The set of alternative plans at time t is denoted by $\Lambda(t)$.

Summary measures of draft alternatives are shown in Table 1. With a decrease of pupils, minimum number of schools also decreases. Possible alternatives, however, do not always decrease with the minimum number of schools. This is because there are more combinations of choosing 9 schools than 11 ones from existing 16 schools. Accessibility works as a strong constraint while capacity is not effective at all. As the distribution of pupils becomes sparser, relative strength of accessibility constraint becomes higher.

Table 1 Summary measures of draft alternatives in Inage ward.

	2010	2030	2050
Number of pupils	6984	5224	4022
Minimum number of schools ($M_{\min}(t)$)	11	9	7
Possible number of alternatives ($\#(\Lambda_M(t))$)	4368	11440	11440
Number of alternatives under constraints ($\#(\Lambda(t))$)	521	179	85
Flexibility of location planning ($\Phi(\Lambda(t))$)	2.717	2.253	1.930
Strength of efficiency constraint ($\Delta\Phi(\Lambda_M(t), \Lambda_0(t))$	1.176	0.758	0.758
Strength of accessibility constraint ($\Delta\Phi(\Lambda_A(t), \Lambda_M(t))$	0.923	1.806	2.129
Strength of capacity constraint ($\Delta\Phi(\Lambda_C(t), \Lambda_M(t))$	0.000	0.000	0.000

School buildings are designed especially for elementary education. Consequently, it is quite difficult to use for other purposes. It is also quite inefficient to close them for a certain period of time. We thus solve problem P_4 mentioned in Section 5.4. We choose P_4 instead of P_3 because of its high tractability.

Solving the optimization problems independently in 2010, 2030 and 2050, we obtain 521, 179, and 85 independent alternatives. They compose 521*179*85=7927015 spatiotemporal plans. From them we choose 2762 alternatives that satisfy the constraint defined by equation (23). Consequently, the strength of continuity constraint is

$$\log(7927015) - \log(2762) = 3.458. \tag{37}$$

Effect of continuity constraint on facility location is evaluated also for each school. It is measured individually in 2010, 2030, and 2050. Figure 9 shows the distribution of demand by continuity constraint. As seen in the figure, schools of high continuity constraint are either those of high accessibility or capacity constraint. Schools of low accessibility and capacity constraints have low continuity constraint. This is quite a reasonable result.

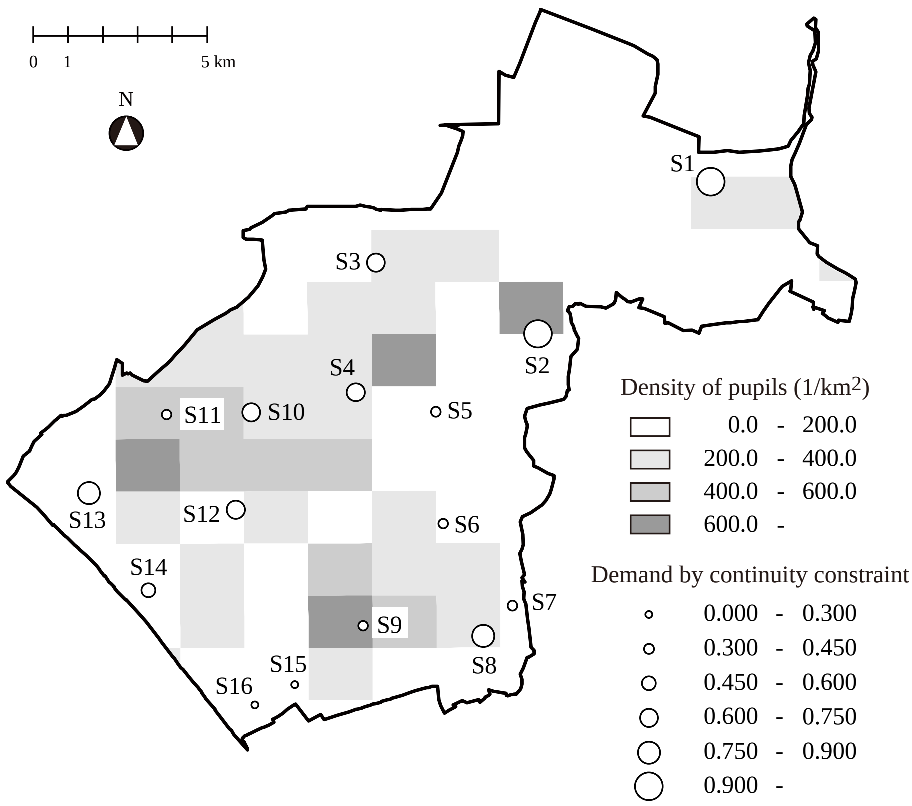

Figure 9 Demand by continuity constraint. Gray shades indicate the density distribution of children aged 6-12 in 2050.

We finally discuss political options of school location planning in Inage ward. The above result clearly shows that almost half of existing schools should be closed until 2050 to attain economic efficiency. We thus focus on how to choose schools to be closed.

We start with the location of schools in 2050. Figure 4 indicates the absolute necessity of individual schools in 2050. In this figure we notice that schools S1 and S3 are almost indispensable. They are necessary due to accessibility constraint as shown in Figure 5. However, since they are located in the suburban area of Inage ward, they are not filled with many pupils as shown in Figure 2. Consequently, an efficient option it to introduce a school bus system in this area covering schools S1, S2, and S3 that have high accessibility constraint measure. This permits us to keep one of S1, S2, and S3 and close the others.

In Figure 4, necessity of other schools is relatively low. Their accessibility constraint is relatively weak, and they are closely located with each other. Consequently, it is enough to focus on the capacity constraint of the remaining schools.

Considering the spatial distribution of schools, we divide them into three groups: G1={S4, S5, S10, S11}, G2={S12, S13, S14}, G3={S6, S7, S8, S9, S15, S16}. In G1, schools have relatively many pupils. They fill almost half of the capacity of each school. Consequently, it would be reasonable to keep two among four schools in this group. G2 schools, on the other hand, have fewer pupils so that it is enough to keep only one school and close the others. G3 schools also have few pupils. Two schools would be enough to serve all the pupils in this area.

To choose schools in each group, it is useful to consider the uncertainty of population distribution in future. Schools of higher continuity constraint should be chosen rather than those of lower one. Figure 9 suggests {S4, S5}, {S13}, and {S8, S16} in G1 G2, and G3 groups, respectively.

The above discussion suggests that six schools would be enough in 2050. It is fewer by one than that obtained by solving the spatial optimization, because the introduction of bus system is considered. As seen in Figure 8, accessibility constraint is very restrictive in Inage ward. Introduction of school bus system or other public transportation system would greatly increase the flexibility of school location, and consequently, leads to efficient management of educational system.

7. Conclusion

This paper proposes a new method of decision support of spatiotemporal facility location. It consists of two steps, that is, preparation and evaluation of draft plans. The former partly utilizes spatial optimization technique. Draft plans are evaluated in several aspects by using quantitative measures. Flexibility of facility location and strength of constraints are key concepts in evaluation. The proposed method was applied to a school planning in Inage ward, Japan. The result revealed the properties of the method as well as provided empirical findings.

We finally discuss the limitations and extensions of the proposed method for future research. First, more empirical studies are indispensable to evaluate and improve the method. Effectiveness of method heavily depends on the given setting that includes the location of existing facilities, the distribution of facility users, and their change over time. Since Inage ward is rather homogeneous, whether the method works effectively should be validated in different settings. Second, since this paper assumes population decrease, it considers only the closure of existing facilities. However, even in population decrease, some facilities become more necessary and important. For instance, population decrease often involves with an increase of aged people. This raises the demand for facilities for aged people including hospitals, elderly day care centers, food delivery services, and so forth. Facility location should be considered with respect to not only decrease but also increase in demand for the service of facilities. Third, variation among facilities should be taken into account in evaluation of facility location. Public facilities are generally homogeneous compared with commercial facilities such as retail shops and restaurants. However, a variation usually exists even among the same type of public facilities, typically in size and functions. Such a variation affects the evaluation of facilities and choice behavior of their users. Further refinement of the method is an important subject in future research.

References

Daskin, M. S. (1995): *Network and Discrete Location: Models, Algorithms, and Applications*. Wiley.

Dodge, M., McDerby, M. and Turner, M. (2008): *Geographic Visualization: Concepts, Tools and Applications*. Wiley.

Drezner, Z. (1995): *Facility Location: A Survey of Applications and Methods*. Springer.

Drezner, Z. and Hamacher, H. W. (2004): *Facility Location: Applications and Theory*. Springer.

Farr, D. (2007): *Sustainable Urbanism: Urban Design with Nature*. Wiley.

Healey, P. (1997): *Collaborative Planning: Shaping Places in Fragmented Societies*. Palgrave Macmillian.

Iwamoto, K. (2010): *Spatiotemporal Location Planning of Public Facilities*. Graduation Thesis, Department of Urban Engineering, University of Tokyo.

Kraak, M.-J. and Ormeling, F. J. (2009): *Cartography: Visualization of Spatial Data*. Prentice-Hall.

Langner, M. and Endlicher, W. (2007): *Shrinking Cities: Effects on Urban Ecology and Challenges for Urban Development*. Peter Lang Publishing.

MacEachren, A. M. (2004): *How Maps Work: Representation, Visualization, and Design*. Guilford Press.

Mirchandani, P. B. and Francis, R. L. (1990): *Discrete Location Theory*. Wiley.

Saaty, T. L. and Peniwati, K. (2007): *Group Decision Making: Drawing Out and Reconciling Differences*. RWS Publications.

Sadahiro, Y. and Sadahiro, S. (2008): A Decision Support Method for Facility Location Planning in Population Decrease. *Discussion Paper* **100**. Department of Urban Engineering, University of Tokyo.

Wood, D. (1992): *The Power of Maps*. Guilford Press.