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Decision support of dynamic facility location in population decrease

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Abstract

Population decrease is one of the most critical issues in urban planning in developed countries. This requires us the closure, integration and size reduction of public facilities for economic efficiency. This paper proposes a new decision support method of dynamic facility location in population decrease. A focus is on providing a knowledge basis for participants of collaborative planning to understand the properties of facility location problem that they are facing. Specifically, the relationship between the flexibility of facility location and necessary conditions is analyzed. The properties of facility location problem are described by quantitative measures and visualized by tables, figures and maps. The method is applied to the school reduction planning in Inage Ward, Japan. It reveals the properties of the method as well as provides empirical findings.

Keywords: *decision support, dynamic facility location, population decrease, necessary conditions*

1. Introduction

Population decrease is one of the most critical issues in urban planning in developed countries (Farr, 2007; Langner and Endlicher, 2007). Local communities collapse and social capital decreases. Retail stores are unable to continue their business due to a sharp decrease in sales. Local governments are burdened with the cost of providing public services.

Location of public facilities has to be reconsidered to meet this new phase. Population decrease requires the closure, integration and size reduction of existing facilities with a long-term view. Dynamic facility location planning has to be discussed by politicians, administrators, local communities, private companies, and non-profit organizations (Healey, 1997; McCarthy and Lloyd, 2007; Saaty, and Peniwati, 2007; Kiminami *et al.*, 2009).

Since public facility planning involves various sectors of population, group discussion and collaborative decision making are essential. Visual aids such as tables, figures and maps are useful to share a common knowledge among participants and to reach a final agreement. Especially, maps are effective tools to analyze spatial phenomena because human eyes are powerful detectors of spatial patterns (Wood, 1992; MacEachren, 2004; Dodge *et al.*, 2008; Kraak and Ormeling, 2009). If we compare the distributions of population and facilities, we may find areas where facilities can be reduced to serve the same demand. Choropleth maps showing the number of facilities per person are useful to evaluate the service level of public facilities at a local scale.

Though visual aids play an important role in facility location planning, discussion only based on visual analysis often becomes rather vague and subjective. The final decision is not always persuasive enough for all the participants.

Another option is to utilize spatial optimization technique that mathematically gives an optimal location of facilities (Mirchandani and Francis, 1990; Drezner, 1995; Drezner and Hamacher, 2004). It calculates the location of facilities that is optimal in a certain aspect such as the operation cost of facilities, travel cost of facility users, and so forth. Necessary conditions are represented as constraints, say, the capacity of facilities and the maximum travel distance of facility users.

One drawback of spatial optimization is that it highly abstracts the real world. Homogeneity is assumed throughout the model, from the properties of facilities to the preference of their users. Such abstraction is not easily acceptable in a practical sense, and consequently, facility location derived from spatial optimization is often unrealistic and infeasible. In addition, spatial optimization assumes an objective and quantitative representation of spatial phenomena. It thus suffers from the vagueness and uncertainty in human evaluation and behavior. The facility location obtained from spatial optimization is not necessarily optimal in the real world.

To resolve the above problems, this paper proposes a new decision support method of dynamic facility location. A fruitful discussion requires understanding of the properties of the problem that we are facing. It is essential to grasp the present status of facility location, to understand the difficulties of the problem, and to evaluate possible alternatives. This paper aims to support collaborative facility planning by providing a knowledge basis for participants to understand and discuss the properties of their problem.

Discussion starts with a static facility location planning, followed by its extension to a dynamic planning by incorporating the temporal dimension. Section 2 outlines the general setting of the method. In this setting, Section 3 proposes two methods of preparing draft plans used for understanding of the properties of facility location problem. The draft plans are analyzed and their properties are described by quantitative measures in Section 4. In the following two sections, the proposed method is extended to a dynamic facility location planning and applied to the school reduction planning in Inage Ward, Japan. Section 7 summarizes the conclusions with discussion.

2. General setting

Suppose that facilities and their users are distributed in a two-dimensional region. Let F_i (i=1, 2, ..., M) and U_j (j=1, 2, ..., N) be the ith facility and jth user, respectively. All the facilities provide the same service at the same level. Distance between facility F_i and user U_j is represented by d_{ij}.

Population decrease requires the closure of existing facilities for the improvement of economic efficiency. Let Ω be a draft plan of facility location in future. It indicates whether each facility is kept open or closed by a binary function:

$$f_i(\Omega) = \begin{cases} 1 & \text{if } F_i \text{ remains open in } \Omega \\ 0 & \text{otherwise} \end{cases}. \tag{1}$$

Draft plans determine not only the location but also the capacity and accessibility of facilities. The capacity of facility F_i and the accessibility of facility F_i to user U_j are represented as $c_i(\Omega)$ and $a_{ij}(\Omega)$, respectively. The latter is typically defined as

$$a_{ij}(\Omega) = \begin{cases} 1 & \text{if } d_{ij} \le d_{\max}(\Omega) \\ 0 & \text{otherwise} \end{cases}, \tag{2}$$

where users go to facilities only located within distance $d_{\max}(\Omega)$.

3. Preparation of draft plans

In the above setting, we prepare draft plans of facility location. Unlike usual collaborative planning, however, this paper uses the plans as a basis for understanding the properties of facility location problem rather than as feasible options from which the final plan is chosen.

In facility location problems, individuals usually wish the final plan to meet their own demand. We call it a *condition* because it works as a determinant of facility location by limiting the available options. This paper considers three conditions in preparing draft plans: the *reduction, accessibility* and *capacity condition*. The first condition claims that existing facilities should be reduced as much as possible for economic efficiency. The second one is that every user should have at least one accessible facility. The third one insists that the number of users should not exceed the capacity of facilities. These conditions limit alternative plans, either strictly or loosely.

3.1 Stochastic method

One method of generating draft plans is based on the necessity of existing facilities. We assume a random choice model of facility users where every user chooses a facility from accessible ones with the same probability. The number of facilities accessible to user U_j in plan Ω is

$$m_j(\Omega) = \sum_i a_{ij}(\Omega). \tag{3}$$

The probability of user U_j choosing F_i is

$$\gamma_{ij}(\Omega) = \frac{1}{m_j(\Omega)}. \tag{4}$$

This model permits us to evaluate the necessity of each facility in two aspects, that is, accessibility to users and capacity of facilities. If users have many accessible options, facility choice is flexible so that the necessity of each facility is relatively low. This is represented by the *necessity of* F_i *by accessibility condition* in plan Ω defined as

$$\nu_i^A(\Omega) = \max_j \gamma_{ij}(\Omega). \tag{5}$$

This measure indicates the highest probability of a facility being chosen by its users. It takes one if a facility has at least one user of only one accessible facility, even though the other users have many options. This typically happens in rural areas where facility users have fewer options. Lower boundary of $\nu_i^A(\Omega)$ is zero, which appears when many facilities are accessible to all the users.

The necessity of facilities also depends on their capacity compared to the number of their potential users. A facility is necessary if it is expected to have many users. The *necessity of* F_i *by capacity condition* in plan Ω is defined as

$$\nu_i^C(\Omega) = \min\left\{\frac{n_i(\Omega)}{c_i(\Omega)}, 1.0\right\}. \tag{6}$$

The first term is the *occupation rate*, the ratio of the expected number of users to the capacity of facility. The second term sets the upper limit of $\nu_i^C(\Omega)$ to one to make it comparable to $\nu_i^A(\Omega)$.

This implicitly claims that a facility is indispensable if it has more users than its capacity.

The above two necessities define the overall necessity of a facility. The *absolute necessity of* F_i in plan Ω is given by

$$\nu_i(\Omega) = \max\left\{\nu_i^A(\Omega), \nu_i^C(\Omega)\right\}. \tag{7}$$

A large $\nu_i(\Omega)$ implies that F_i is highly necessary for users in its neighborhood. On the other hand, a facility of a small $\nu_i(\Omega)$ is a candidate to be closed for economic efficiency.

The stochastic method generates draft plans of facility location by using a stochastic process where the probability of facility F_i remaining open is given by its necessity $\nu_i(\Omega)$. Let Ω_0 be a draft plan that keeps the present status of facilities and their accessibility. Given the number of draft plans, the stochastic method generates them according to the multinomial distribution of parameter $\nu_i(\Omega_0)$.

A strength of the stochastic method is its simplicity. Its theoretical background is easy to understand and implementation is straightforward. The method assumes that facilities of high necessity are more likely to remain open, while those of low necessity tend to be closed. Though the underlying model is rather too simple, the stochastic method is a practical option to generate draft plans.

3.2 Optimization method

Another method to prepare draft plans is to utilize the spatial optimization technique that gives an optimal location of facilities (Mirchandani and Francis, 1990; Daskin, 1995; Drezner, 1995; Drezner and Hamacher, 2004). It minimizes the number of facilities under accessibility and capacity constraints, that is, every user is assigned one accessible facility and the number of users assigned to a facility does not exceed its capacity. This problem is formulated as

$$\min_{f_i(\Omega), x_{ij}(\Omega)} \sum_i f_i(\Omega), \tag{8}$$

subject to

$$\begin{aligned} \sum_j x_{ij}(\Omega) &\le c_i, \forall j \\ x_{ij}(\Omega) &\le f_i(\Omega), \forall i, j \\ x_{ij} a_{ij}(\Omega) &= 1, \forall i, j \\ \sum_j x_{ij}(\Omega) &= 1, \forall j \end{aligned}, \tag{9}$$

where $x_{ij}(\Omega)$ is a binary function representing the assignment of user U_j to facility F_i.

Solving the above problem, we obtain the minimum number of facilities $M_{\min}$ and a set of facility locations that minimize the number of facilities. Note, however, $M_{\min}$ is achieved by not only a single set of locations. There can be numerous plans that gives $M_{\min}$, all of which are candidates for draft plans. We thus generate draft plans by changing the values of f_i step by step from the initial set of facility location (for details, see Sadahiro and Sadahiro, 2009).

A difference between the stochastic and optimization methods lies in the strictness of conditions. The former regards the conditions as desirable ones while the latter uses them as strict constraints that must be satisfied in facility location.

A strength of the optimization method is that it always satisfies the given conditions. On the other hand, since the optimization problem is rather complicated, it often takes a long time to find a solution. Moreover, since the problem is solved by a heuristic method, it is not assured to reach the global optimal solution. The choice between the two methods depends on the strictness of conditions, computational environment, and so forth.

4. Analysis of draft plans: Toward understanding of the properties of facility location problem

Having obtained draft plans, we analyze them to understand the properties of facility location problem we are facing. Draft plans are evaluated on both individual and group basis, in each of which individual facilities are also evaluated.

4.1 Evaluation of facilities

Given a draft plan, either by the stochastic or optimization method, we evaluate the desirability of individual facilities. Basic measures include the occupation rate and the average distance between users and facilities. When draft plans are generated by the stochastic method, the measures are given by

$$n_i(\Omega) = \frac{\sum_j \gamma_{ij}(\Omega)}{c_i} \tag{10}$$

and

$$\bar{d}_i(\Omega) = \frac{\sum_j \gamma_{ij}(\Omega) d_{ij}}{n_i(\Omega)}, \tag{11}$$

respectively. If the optimization method is adopted, the measures are given by

$$n_i(\Omega) = \frac{\sum_j x_{ij}(\Omega)}{c_i}$$

(12)

and

$$\bar{d}_i(\Omega) = \frac{\sum_j x_{ij}(\Omega) d_{ij}}{n_i(\Omega)},$$

(13)

respectively.

4.2 Evaluation of draft plans

To evaluate draft plans, we propose two concepts called the *perspective* and *score*. The perspective, denoted by Λ, is a framework of evaluating draft plans. It examines draft plans in various aspects such as the practical feasibility, the degree of satisfying the conditions, and so forth. The score, on the other hand, is a summary measure of overall evaluation of a draft plan. Given a perspective, we evaluate draft plans and describe the result as a score. The perspective is, in a sense, a representation of sense of values, which is quantitatively described by a set of scores.

Let us consider the evaluation of draft plans in concrete examples. Given *M* facilities, we have 2^M possible combinations of facility location $\{\Omega_1, \Omega_2, ..., \Omega_{2^M}\}$. The null perspective where all the plans are equally desirable is denoted by Λ_0. In this perspective, the score of each plan is constant:

$$s(\Omega_k, \Lambda_0) = K.$$

(14)

Let Λ_S be the perspective of the stochastic method. In this perspective, the score of a draft plan is defined based on the absolute necessity of individual facilities:

$$s(\Omega_k, \Lambda_S) = \prod_i \left[f_i(\Omega_k) v_i(\Omega_0) + \{1 - f_i(\Omega_k)\}\{1 - v_i(\Omega_0)\} \right].$$

(15)

Let us move to the optimization method. Draft plans generated by the optimization method can be evaluated in various ways. Let Λ_M be the perspective of the optimization method where all the plans giving the minimum number of facilities $M_{\min}$ are equally desirable. In this perspective, the score of these plans is a positive constant while that of the others in $\{\Omega_1, \Omega_2, ..., \Omega_{2^M}\}$ is zero:

$$s(\Omega_k, \Lambda_M) = \begin{cases} K & \text{if } \sum_i f_i(\Omega_k) = M_{\min} \\ 0 & \text{otherwise} \end{cases}.$$

(16)

The draft plans can also be evaluated in other aspects such as the average distance between facilities and their users. In this case, the score may be defined as the inverse of the average distance:

$$s\left(\Omega_k, \Lambda_{M'}\right) = \begin{cases} \dfrac{M_{\min} n_i\left(\Omega_k\right)}{\sum_i \sum_j x_{ij}\left(\Omega_k\right) d_{ij}} & \text{if } \sum_i f_i\left(\Omega_k\right) = M_{\min} \\ 0 & \text{otherwise} \end{cases}. \tag{17}$$

We should note that the score is a relative measure of overall evaluation of a draft plan. It is comparable only within the same perspective, and is enough for the objective of this paper. To compare the score between different perspectives, it has to be standardized across the perspectives. Since the standardization is out of the scope of this paper, we leave it as a topic for future research.

4.3 Evaluation of perspectives

In facility location problems, some conditions are strong enough to be called requirements or constraints, while others are so weak that they do not have to be necessarily satisfied. Conditions are the determinants of facility location, either strong or weak, whose analysis is indispensable to understand the properties of facility location problem.

In this section, we analyze the perspective of evaluating draft plans where the effect of conditions is most reflected. To measure the effect of conditions, we focus on the flexibility of facility location. A high flexibility provides us many options, and consequently, increases the probability of an agreement being reached. If a condition is found to narrow options severely, we may have to consider its relaxation such as the improvement of accessibility and the expansion of existing facilities.

We assume a stochastic process in the choice of a final plan from draft candidates. The probability of plan Ω_k being chosen is proportional to its score:

$$P\left(\Omega_k, \Lambda\right) = \frac{s\left(\Omega_k, \Lambda\right)}{\sum_l s\left(\Omega_l, \Lambda\right)}. \tag{18}$$

The *flexibility* of perspective Λ is measured by the entropy of the stochastic process:

$$\begin{aligned} \Phi\left(\Lambda\right) &= -\sum_k P\left(\Omega_k, \Lambda\right) \log P\left(\Omega_k, \Lambda\right) \\ &= -\frac{1}{\sum_l s\left(\Omega_l, \Lambda\right)} \sum_k s\left(\Omega_k, \Lambda\right) \left\{ \log s\left(\Omega_k, \Lambda\right) - \log \sum_l s\left(\Omega_l, \Lambda\right) \right\} \end{aligned}. \tag{19}$$

Substituting equation (14), we obtain the flexibility of Λ_0:

$$\Phi\left(\Lambda_0\right) = M \log 2.$$

Comparing $\Phi(\Lambda)$ with $\Phi(\Lambda_0)$, we can evaluate the effect of conditions incorporated in perspective Λ:

$$\Delta\Phi\left(\Lambda,\Lambda_0\right)=\Phi\left(\Lambda_0\right)-\Phi\left(\Lambda\right). \tag{20}$$

A large value indicates that the conditions greatly limit the options of facility location.

The effect of individual conditions can be evaluated separately. Let us suppose two stochastic methods in each of which either accessibility or capacity condition is considered. Their perspectives are denoted by Λ_{SA} and Λ_{SC}, respectively. The effects of accessibility and capacity conditions are measured by

$$\Delta\Phi\left(\Lambda_{SA},\Lambda_0\right)=\Phi\left(\Lambda_0\right)-\Phi\left(\Lambda_{SA}\right) \tag{21}$$

and

$$\Delta\Phi\left(\Lambda_{SC},\Lambda_0\right)=\Phi\left(\Lambda_0\right)-\Phi\left(\Lambda_{SC}\right), \tag{22}$$

respectively.

The effect of conditions can also be evaluated for individual facilities. Let $p(F_i;\Lambda)$ be the probability that F_i is chosen to remain open in the final plan. The flexibility of facility F_i is measured by

$$\phi\left(F_i;\Lambda\right)=p\left(F_i;\Lambda\right)\log p\left(F_i;\Lambda\right)+\left\{1-p\left(F_i;\Lambda\right)\right\}\log\left\{1-p\left(F_i;\Lambda\right)\right\}. \tag{23}$$

As seen in the equation, the flexibility is maximized when $p(F_i;\Lambda)$ is 0.5, that is, whether F_i is kept open or closed is equally likely to happen.

If we add a new condition, the probability $p(F_i;\Lambda)$ changes. The effect of this condition thus measured by

$$\Delta\phi\left(F_i;\Lambda',\Lambda\right)=\phi\left(F_i;\Lambda'\right)-\phi\left(F_i;\Lambda\right), \tag{24}$$

where Λ’ denotes the new perspective that includes the new condition. Substituting equation (23) into (24), we obtain

$$\begin{aligned}\Delta\phi\left(F_i;\Lambda',\Lambda\right)&=p\left(F_i;\Lambda'\right)\log p\left(F_i;\Lambda'\right)-p\left(F_i;\Lambda\right)\log p\left(F_i;\Lambda\right)\\&\quad+\left\{1-p\left(F_i;\Lambda'\right)\right\}\log\left\{1-p\left(F_i;\Lambda'\right)\right\}-\left\{1-p\left(F_i;\Lambda\right)\right\}\log\left\{1-p\left(F_i;\Lambda\right)\right\}.\end{aligned} \tag{25}$$

This equation indicates that adding a new condition on F_i has both positive and negative effects on F_i. The first line of the right hand term of equation (25) reflects the positive effect, that is, an increase in the probability of F_i remaining open. The second line, on the other hand, represents the negative side of introducing the new condition, an increase in the probability of being closed. These effects are represented separately as

$$\Delta\phi\left(F_i;\Lambda',\Lambda\right)=\Delta\phi_+\left(F_i;\Lambda',\Lambda\right)+\Delta\phi_-\left(F_i;\Lambda',\Lambda\right), \tag{26}$$

where

$$\Delta\phi_+\left(F_i;\Lambda',\Lambda\right)=p\left(F_i;\Lambda'\right)\log p\left(F_i;\Lambda'\right)-p\left(F_i;\Lambda\right)\log p\left(F_i;\Lambda\right) \tag{27}$$

and

$$\Delta\phi_-\left(F_i;\Lambda',\Lambda\right)=\left\{1-p\left(F_i;\Lambda'\right)\right\}\log\left\{1-p\left(F_i;\Lambda'\right)\right\}-\left\{1-p\left(F_i;\Lambda\right)\right\}\log\left\{1-p\left(F_i;\Lambda\right)\right\}. \tag{28}$$

To discuss the closure of existing facilities, it is enough to evaluate the positive effect of conditions. The effects of accessibility and capacity conditions are evaluated by *demand for* F_i *by accessibility condition* and *demand for* F_i *by capacity condition*:

$$\Delta\phi_+\left(F_i;\Lambda_{SA},\Lambda_0\right)=p\left(F_i;\Lambda_{SA}\right)\log p\left(F_i;\Lambda_{SA}\right)-p\left(F_i;\Lambda_0\right)\log p\left(F_i;\Lambda_0\right) \tag{29}$$

and

$$\Delta\phi_+\left(F_i;\Lambda_{SC},\Lambda_0\right)=p\left(F_i;\Lambda_{SC}\right)\log p\left(F_i;\Lambda_{SC}\right)-p\left(F_i;\Lambda_0\right)\log p\left(F_i;\Lambda_0\right), \tag{30}$$

respectively.

The measures proposed above are visualized as tables, figures and maps to help our understanding of the properties of facility location problem. In addition, they are useful to evaluate the present status of facility location. For instance, a small $\Delta\phi_+(F_i;\Lambda_{SC},\Lambda_0)$ and a large $\Delta\phi_+(F_i;\Lambda_{SA},\Lambda_0)$ imply that facility F_i has fewer users than its capacity but still necessary due to the lack of accessibility. If this happens on many facilities, it is effective not to treat the accessibility condition as a given requirement but to consider political options to improve the accessibility such as the introduction of public transport systems. Let us then suppose another case where two neighboring facilities show a large and a small $\Delta\phi_+(F_i;\Lambda_{SC},\Lambda_0)$, respectively. This implies that the former is surrounded by many users while the latter has fewer users in its neighborhood. In this case we may integrate them into the former by its expansion to serve the users of both facilities.

5. Dynamic facility location planning

So far we have discussed the facility location planning as a static problem. This section extends the proposed method to the dynamic domain, that is, a dynamic facility location problem during a certain period of time.

There are at least three properties specific to the dynamic facility location problem: 1) continuity of temporal dimension, 2) uncertainty in future prospect, and 3) duration condition. They are discussed with respect to the preparation of draft plans and the analysis of draft plans in the following.

5.1 Preparation of draft plans

In the static facility location problem discussed so far, a limited number of locations are given a priori as possible locations of facilities. Consequently, it is enough to discuss the facility location in a discrete space. Dynamic facility location problem, on the other hand, has to be considered in a continuous space because the temporal dimension is continuous. The continuity as well as multidimensionality increases the complexity and difficulty of the problem.

A practical approach to prepare draft plans in dynamic facility location planning is to approximate the temporal dimension by a discrete space. We can derive draft plans at sections in a time period separately and combine them into one as an approximation of a dynamic plan.

Uncertainty is another property specific to dynamic facility location problem. Future prospect inevitably involves uncertainty and ambiguity. Facility location should be flexible enough to deal with unexpected distribution of facility users.

One method to treat the uncertainty is to apply smoothing operation to the expected distribution of facility users. This yields a continuous surface that approximates possible locations of users in future. Using the surface to generate draft plans, we can avoid extreme plans that cannot cope with unexpected change in user distribution.

The third property indicates that facilities cannot be opened, closed or moved flexibly and immediately to meet the continually changing demand. It is possible only if facilities are designed for general purposes and the cost of immediate opening and closure is very low. In usual, once a facility is opened or closed, the same status has to continue for a while. We call this a *duration condition*, which is specific to dynamic facility location problem.

A simple method to take this condition into account is to prepare draft plans at time sections and extract their combinations that satisfy the duration condition. Though this method does not work efficiently, its implementation is straightforward.

A more sophisticated method is to include the duration condition in the constraints in the optimization method. The dynamic facility location problem is solvable by heuristic methods similar to the static facility location problem.

5.2 Analysis of draft plans

The measures proposed in Section 4 are also effective in dynamic facility location. We calculate them at time sections and summarize them as basic statistics such as the mean and variance over a time period.

In a practice sense, it is useful to evaluate the effect of duration condition on facility location. It is measured by a decrease in the flexibility of facility location caused by adding the duration condition. If the effect is highly restrictive, we may consider the design of general-purpose facilities or the introduction of temporary and portable facilities.

A special case of duration condition is the *closure condition*, which prohibits the reopening of closed facilities. This condition is also practically important because the cost of temporary closure is not negligibly small. If its effect is restrictive, the cost reduction is necessary to increase the flexibility of facility location.

6. Empirical study: School reduction in Japan

This section applies the proposed method to the school reduction planning in Inage Ward in Chiba City, Japan. Inage Ward is located 30 kilometers away in an eastern suburb of Tokyo. There are two railway stations in the south end around which retail stores and restaurants are clustered. The shopping districts are surrounded by densely inhabited urban area. The north is a suburban area mostly covered with residential districts of rather low density.

Inage Ward has 16 public elementary schools in 2010 (Figure 1). They are densely located in urban area in the south, while sparsely distributed in the north. In 2010, there are 6984 pupils in Inage Ward. With a rapid decrease in birth rate, however, pupils have been decreasing since 1981. They are expected to decrease to 5224 and 4022 in 2030 and 2050, respectively. In 2050, one school has only 250 pupils on average, which is too small compared with 720, the size of elementary schools considered as desirable in Japan. School reduction is indispensable to keep educational environment of schools and economic efficiency of educational finance.

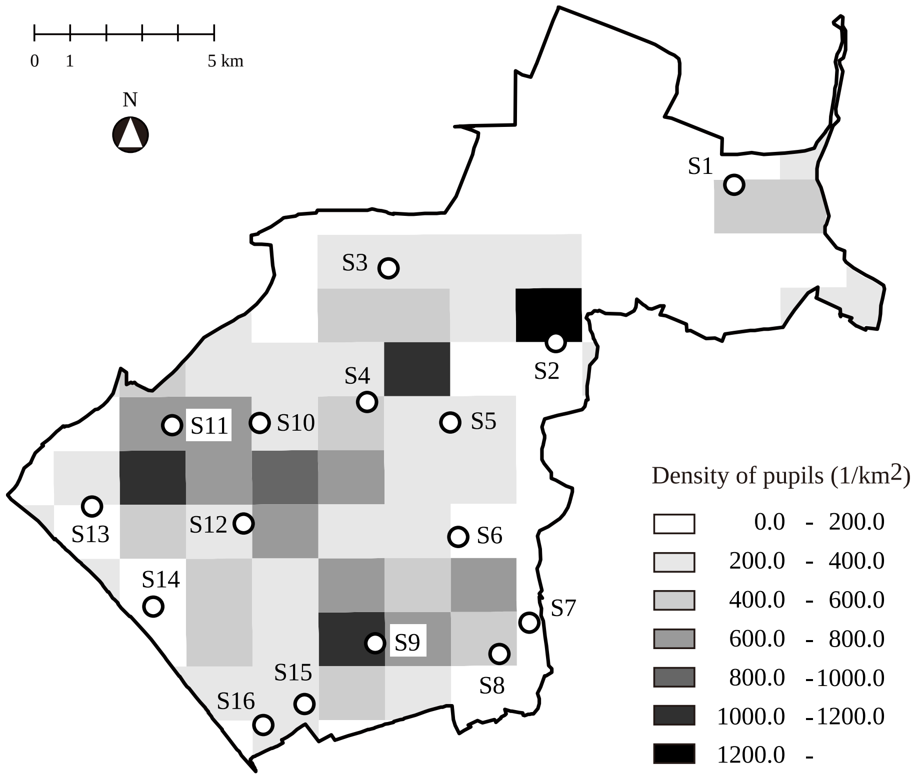

Figure 1 Public elementary schools and the density distribution of pupils in 2010 in Inage Ward, Japan.

We prepared draft plans for 2010, 2030, and 2050 separately by using both the stochastic and optimization methods without the duration condition. For the capacity of schools and the accessibility to pupils, we adopted $c_i(\Omega)$=720 and $d_{\max}(\Omega)$=2km according to the standards provided by the Ministry of Education, Culture, Sports, Science and Technology. Pupils usually go to school by walk in Japan due to its safe environment and short distance from home to school. School bus system is utilized only in limited rural areas.

6.1 Analysis of draft plans generated by the stochastic method

The stochastic method generated 200 draft plans for each year. We calculated the average occupation rate of schools, average distance from home to school, and demands by accessibility and capacity conditions in 2010, 2030, and 2050. Their summary statistics and spatial distributions in 2050 are shown in Table 1 and Figures 2-5.

Table 1 Summary measures of draft plans generated by the stochastic method.

	2010	2030	2050
Average occupation rate of schools	0.607	0.454	0.350
Average distance from home to school	1181	1181	1181

Demand by accessibility condition	1.391	1.391	1.391
Demand by capacity condition	0.607	0.454	0.349
Flexibility of facility location	8.450	9.091	9.092

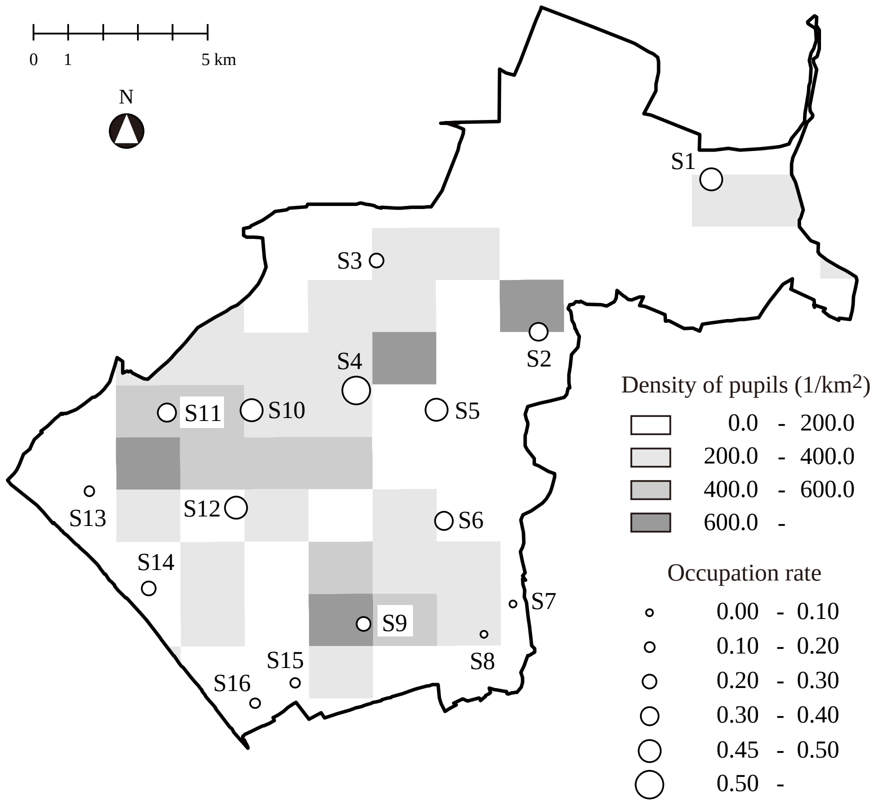

Figure 2 Occupation rate of elementary schools in 2050. Gray shades indicate the density distribution of pupils in 2050.

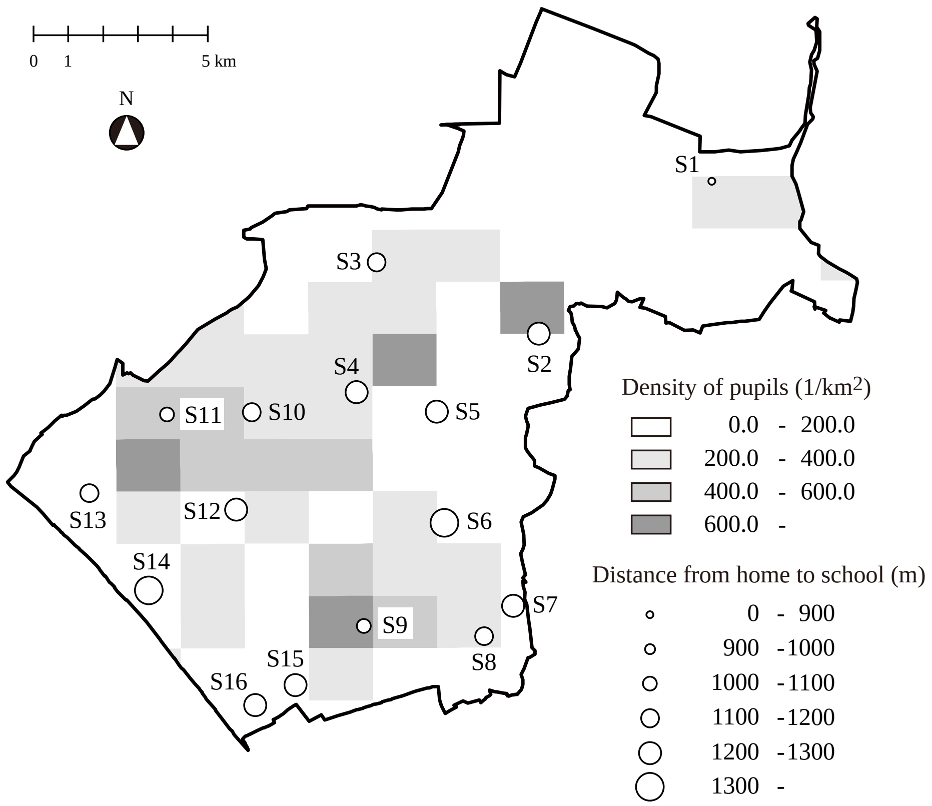

Figure 3 Average distance from home to school in 2050. Gray shades indicate the density distribution of pupils in 2050.

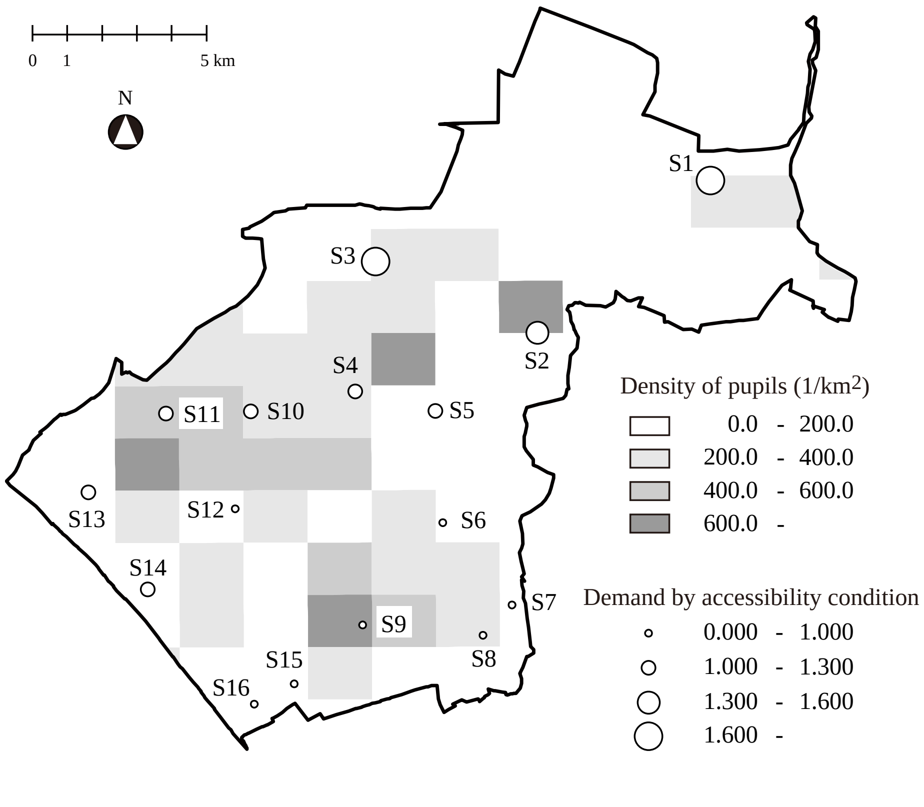

Figure 4 Demand by accessibility condition in 2050. Gray shades indicate the density distribution of pupils in 2050.

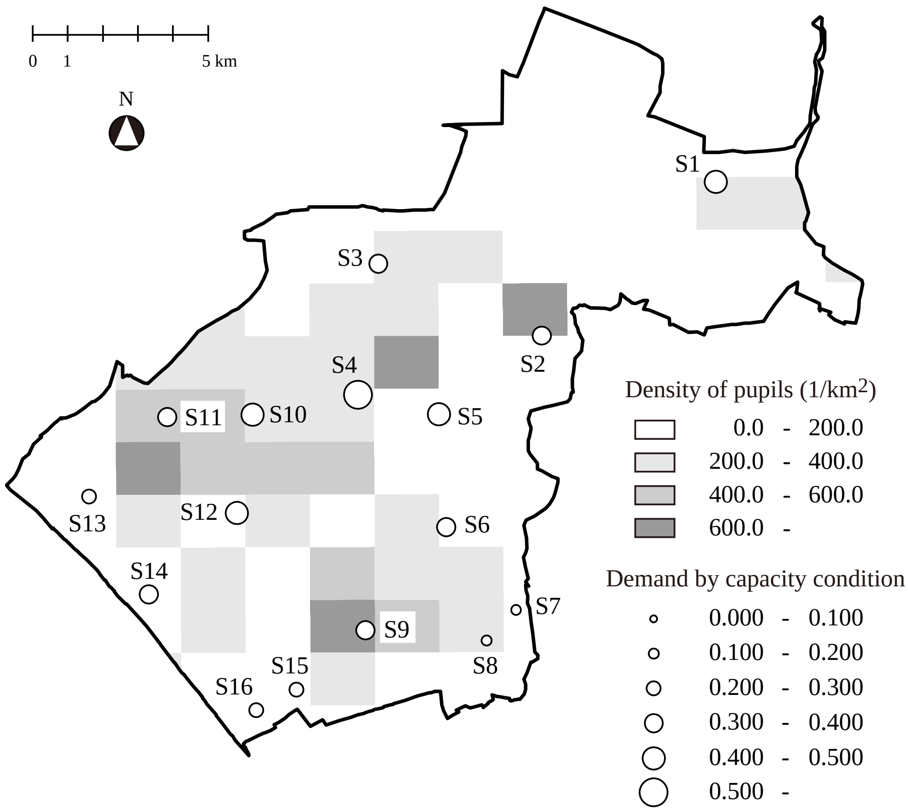

Figure 5 Demand by capacity condition in 2050. Gray shades indicate the density distribution of pupils in 2050.

Table 1 shows the summary measures of draft plans from 2010 to 2050. The occupation rate decreases with the number of pupils, so does the demand by capacity condition. The distance from home to school and demand by accessibility condition are both constant throughout the period. This is because the stochastic method assumes that pupils choose their schools randomly from accessible ones. Pupils may go to distant schools even if closer ones are not full. The flexibility of facility location gradually increases with a decrease in demand by capacity condition.

Using Figures 2-5, we discuss the present status of elementary schools in comparison with the distribution of pupils expected in 2050. As seen in Figure 2, the occupation rate is lower than 0.5 at all the schools except S4. It is prominent at schools in the south such as S7-S9 and S13-S16. School reduction is indispensable for economic efficiency.

Figure 3 may seem rather counterintuitive; the average distance from home to school is larger in urban area than in suburban area. This is also caused by the random choice of schools assumed in the stochastic method. In suburban area pupils are clustered around schools while they are widely spread in urban area. The latter may choose distant schools, which increases the average distance to schools in urban area.

Let us turn to the effect of accessibility and capacity conditions on facility location. Figure 4 shows that the demand by accessibility condition is higher in suburban area, as typically seen in schools S1, S2, and S3. This is because pupils in this area have only a few accessible schools. In contrast, the demand is lower in urban area where pupils have many options.

Figure 5 shows the demand by capacity condition. This figure suggests three different groups of schools, two with high and one with low demands. One group consists of schools in suburban area such as S1, S2, and S3. Their demand is high because they are surrounded by many pupils in their close neighborhood. Another group of high demand includes schools S4, S5, S10, and S12. They are located in urban area in the center of Inage Ward where both schools and pupils are densely distributed. Schools of low demand are observed in urban area in the south. School density is too high compared with that of pupils.

We should note that the demand measures discussed above are calculated based on the relative necessity of facilities, which is not directly connected to the absolute necessity. Consequently, for instance, schools S1, S2, and S3 are relatively important compared with others, but not indispensable because their occupation rate is not high.

So far we have discussed the present status of elementary schools in comparison with the distribution of pupils in 2050. We found that the accessibility condition is critical while the capacity condition is not restrictive. This implies that the improvement of accessibility can increase the flexibility of facility location, which permits us to consider a wider variety of draft plans.

Figure 6 shows the relationship between the accessibility condition and the flexibility of facility location. The flexibility of facility location increases monotonically with the relaxation of accessibility condition, and finally reaches to its maximum. The overall trend is quite similar among all the three cases, though the maximum flexibility slightly varies.

This figure suggests that an improvement of accessibility considerably increases the flexibility of facility location. It increases from 9.5 to 11.0 if the maximum distance from home to school is extended from 2000m to 4000m. This is possible by introducing school bus or other public transport systems.

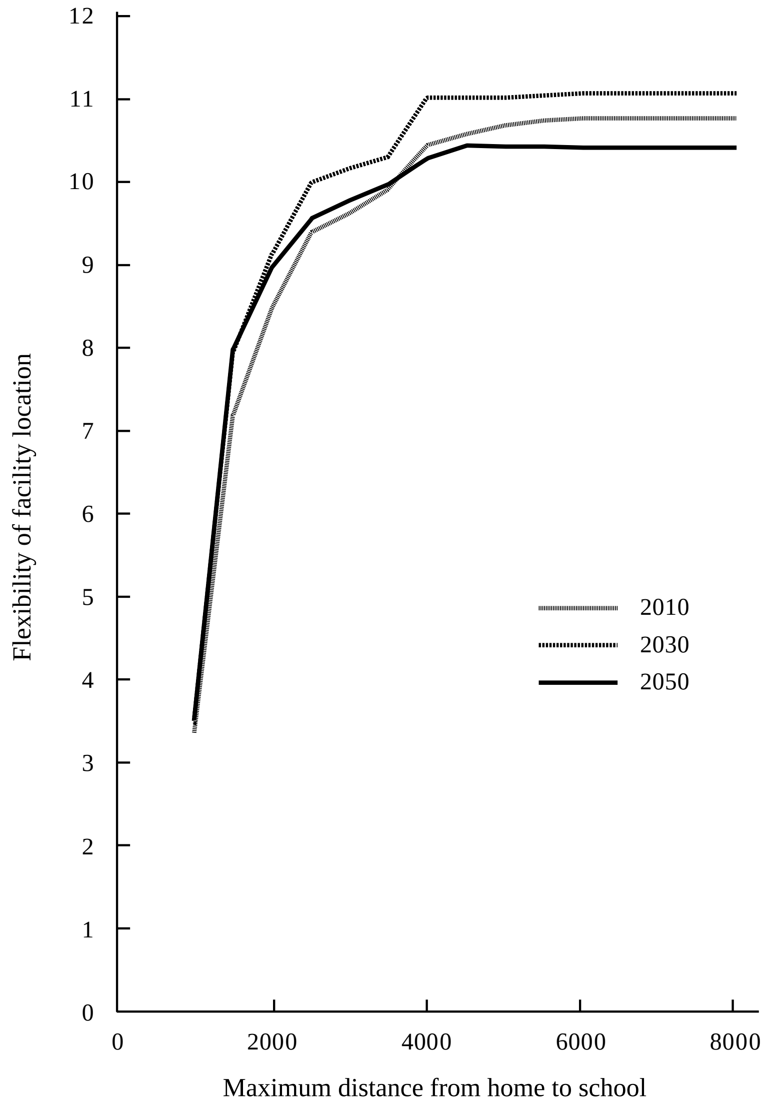

Figure 6 Relationship between the maximum distance from home to school ($d_{max}(\Omega)$) and the flexibility of location planning, from 2010 to 2050.

Figure 7 shows the relationship between the capacity condition and the flexibility of facility location. As capacity increases, the flexibility rapidly increases and then gradually decreases in all the cases. The flexibility reaches its maximum when the average probability of schools being kept open is close to 0.5.

Figure 7 suggests that the capacity of school can be reduced from 720 to 400 in 2050 without decreasing the flexibility of facility location. We can consider size reduction as well as school reduction to improve economic efficiency.

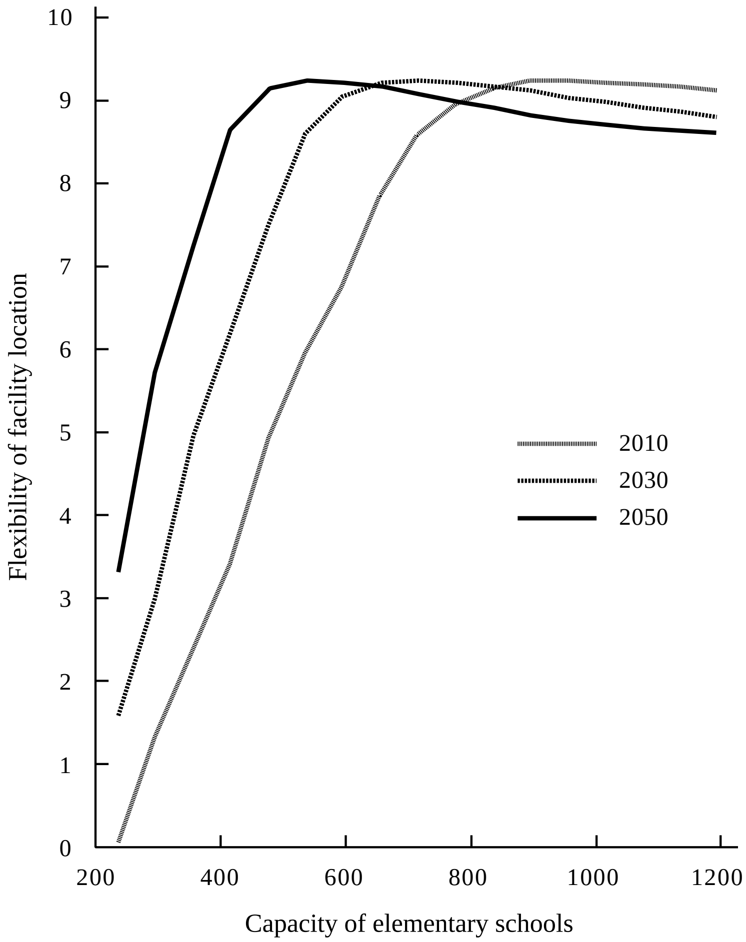

Figure 7 Relationship between the capacity of elementary schools ($c_i(\Omega)$) and the flexibility of location planning, from 2010 to 2050.

6.2 Analysis of draft plans generated by the optimization method

The optimization problem formulated in Subsection 3.2 was solved by a heuristic approach to generate draft plans. The minimum number of elementary schools is 11, 9, and 7 in 2010, 2030, and 2050, respectively. Draft plans were derived as much as possible that minimize the number of schools under the given constraints (for details, see Iwamoto, 2010).

Table 2 shows the summary measures of draft plans from 2010 to 2050. Draft plans decrease with the minimum number of schools, so does the flexibility of facility location. The average occupation rate of schools almost remains the same because pupils and schools decrease in parallel. Demand by capacity condition also stays the same. On the other hand, the school reduction increases the average distance from home to school and the demand by accessibility condition.

As mentioned in Section 3, a difference between the stochastic and optimization methods lies in the strictness of conditions. It is reflected in the flexibility of facility location shown in Tables 1 and 2. Since the optimization method regards the conditions as strict requirements, available options are severely limited, and consequently, the flexibility of facility location is considerably low.

Table 2 Summary measures of draft plans generated by the optimization method.

	2010	2030	2050
Minimum number of schools	11	9	7
Number of draft plans under constraints	521	179	85
Flexibility of facility location	2.717	2.253	1.930
Average occupation rate of schools	0.882	0.806	0.799
Average distance from home to school	1316	1498	1658
Demand by accessibility condition	2.099	2.564	2.887
Demand by capacity condition	0.836	0.788	0.788

6.3 Effect of closure condition

In the above discussion, draft plans were generated without considering the duration condition. This assumes that elementary schools can be opened and closed flexibly to meet the demand. School buildings, however, are designed especially for elementary education in Japan so that the cost of temporary closure is not low.

To take the closure condition into account, we extracted draft plans that satisfy the closure condition from those generated by the optimization method. We obtained 85 plans from 11*9*7=693 options. The closure condition reduces the flexibility of facility location from 2.841 to 1.929.

Since the closure condition is restrictive, its relaxation should be discussed to extend the available options of facility location. Portable and temporary classrooms are effective when other conditions cannot be easily relaxed (Taylor, et al., 1999; Chan, 2009).

6.4 Discussion for school reduction

Using the findings obtained above, we finally discuss the school reduction planning in Inage Ward. The accessibility condition is most critical while the capacity condition is not restrictive. The closure condition is less critical in the sense that it is independent of facilities. In 2050, six schools of capacity 720 is expected enough for serving all the pupils. Our focus is thus on how we reduce existing elementary schools with considering the accessibility condition.

Figure 8 shows the distribution of absolute necessity of schools. As seen in this figure, schools S1 and S3 are most necessary among the existing schools mainly due to the accessibility condition. In the north of Inage Ward, schools S1, S2, and S3 are expected to have 837 (=332+258+247) pupils as a total in 2050. They can be integrated into one school by expanding its capacity and improving the accessibility in this area. It is worth considering the construction of temporary classrooms and the introduction of a school bus system for school integration.

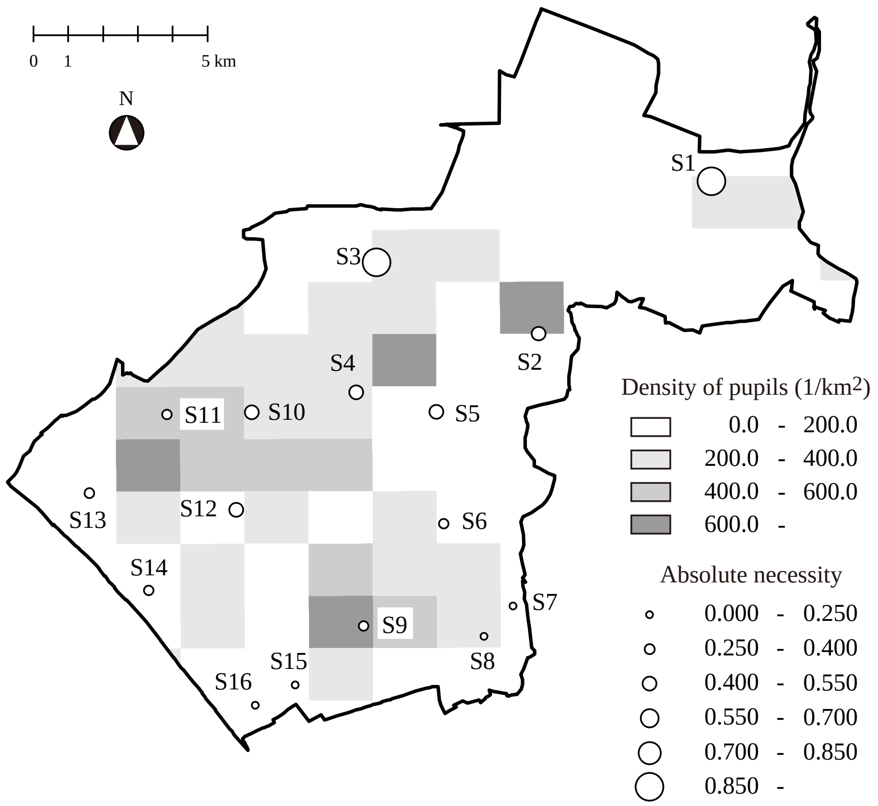

Figure 8 Absolute necessity of elementary schools in 2050. Gray shades indicate the density distribution of pupils in 2050.

In the center and south of Inage Ward, facility location is more flexible because the conditions are not restrictive. To choose schools to remain open, we divide schools into four groups and examine each group separately: G1={S4, S5}, G2={S10, S11, S12, S13, S14}, G3={S6, S7, S8}, G4={S9, S15, S16}. These groups are expected to have 699, 1366, 547, and 574 pupils in 2050, respectively.

In groups G1, G3, and G4, school are neighboring with each other, and one school is enough to serve all the pupils. To choose schools to remain open, it is useful to consider the relative location of schools in each group. In group G1, for instance, school S4 seems better than S5 because the former is located closer to the center of Inage Ward so that it gives a shorter distance from home to school. Similarly, schools S6 and S9 would be better choices in groups S3 and S4, respectively.

In group G2, five schools form a spatial cluster from which two schools should be chosen. Relative location of schools suggests S11 and S12 both of which are located at the center of five schools.

The above procedure is an abstract model of facility location planning. It considers only the accessibility and capacity conditions in choosing schools to remain open. In actual collaborative planning, however, it is obviously necessary to take into account other conditions such as ethnic balance, local communities, safety, and financial support.

7. Conclusion

This paper has developed a new method of decision support of dynamic facility location. Two methods were proposed to generate draft plans used for understanding the properties of facility location problem. An emphasis is on the effect of necessary conditions as the determinants of facility location. The properties are described by quantitative measures and visualized by tables, figures and maps. The proposed method was applied to the school reduction planning in Inage Ward, Japan. The result revealed the properties of the method as well as provided empirical findings.

We finally discuss the limitations and extensions of the method for future research.

First, this paper considers population decrease in facility location. This often involves not only a decrease but also an increase in demand for facilities. In many developed countries, for instance, aged people are increasing while the total population is decreasing. This raises the demand for facilities for aged people including hospitals, elderly day care centers, and food delivery services. Public transport system also becomes necessary because aged people cannot drive by themselves. Location of these facilities should also be discussed in future research.

Second, this paper assumes that all the facilities provide the same type of service at the same level. This gives a high tractability in model-based analysis of facility location problem. This assumption, on the other hand, is not realistic to some extent, even though for public facilities. The proposed method should be refined further by taking into account the variation in the function and service of facilities.

Third, this paper evaluates the effect of conditions mainly in terms of the flexibility of facility location. However, their effect is not limited to the flexibility of facility location. For instance, the desirability and feasibility of facility location are also affected by the conditions. The former is an essential measure of facility location represented as the objective function in spatial optimization. The effect of constraints has been analyzed as sensitivity analysis in existing studies (Drezner, 1985; Labbe *et al*., 1991; Drezner, 1995; Giddings *et al*., 2001). The feasibility of facility location is practically important, though it has not been studied extensively in relation to the necessary conditions. The effect of conditions should be evaluated further in diverse aspects in future research.

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